Elastic Collisions

Content

Before we look at elastic collisions, we are first going to take a step back and look at **kinetic energy** and **momentum**. The **kinetic energy** relates to the energy of motion of an object, it can be calculated using:

$$\vec{E}\_{K}=\frac{1}{2}m\vec{v}^{2}$$

Where $\vec{E}\_{K}$ is the kinetic energy, $m$ is the mass of the object and $\vec{v}$ is the velocity of the object. So, an object that is travelling faster has more kinetic energy than an object of the same mass travelling slower.

Newton described the **momentum** of an object as a quantity of motion. It depends on the mass and motion of the object and can be calculated using the formula:

$$\vec{p}=m\vec{v}$$

When objects collide or crash into each other, we use the kinetic energy and momentum of the system before and after the crash to help understand what happened. An **elastic collision** occurs if the total kinetic energy of the system before and after the crash is the same. To calculate the total kinetic energy of the system, we add the kinetic energy of each object:

$$\sum\_{}^{}\frac{1}{2}m\vec{v}^{2}$$

Where the $Σ$ means to sum, so in this case, we add the kinetic energy for each object together.

As a simple example, consider two balls on a pool table. A red one is approaching a yellow with a constant velocity $\vec{v}$, the yellow one is stationary. The two balls collide and afterwards, the red one is stationary and the yellow one is moving away with the same velocity of the red ball before the collision. Essentially, the balls have switched velocities, so the kinetic energy of the system before the collision and after the collision is the same, making it an elastic collision.

In all collisions, the total momentum of the system is also conserved. Like with kinetic energy, to calculate the total momentum of the system we add the momentum of each object together.

$$\sum\_{}^{}m\vec{v}$$

The total momentum for the system before the crash will *always* be the same as the total momentum after the collision.

Example

Two balls are about to have a head on collision. The first particle has a mass of 1kg travelling with a velocity of 6m/s right. The other has a mass of 0.2kg and is at rest. Their collision is perfectly elastic and the first ball is at rest after the collision while the second is travelling with an unknown velocity. What is the final velocity of the second ball?

Firstly, we begin with a diagram of the system before and after the crash.

Now, we use the **conservation of momentum** and calculate the momentum before and after the crash:



Example

Newton’s cradle is a device with (usually) 5 balls on a held up by a wire just touching each other (see the diagram below). When one ball on the outside is raised and left to fall into the remaining 4 balls, when it crashes into them, it remains motionless while the ball on the other end starts to swing upwards and away from the other balls. This begins a chain reaction where the swinging ball is passed between the two on the end. Using the conservation of momentum and elastic collisions, explain what is happening. What would happen if two balls were initially raised and crashed into the other remaining balls?

* For simplicity, we shall label the balls from left to right as 1-5. So, in the diagram, ball 1 is swinging towards ball two about to crash into it. When ball 1 crashes into ball 2 it remains stationary. Due to conservation of momentum, the momentum of ball 1 must have been transferred into ball 2 (since it cannot be lost) but since ball 2 cannot swing it immediately ‘crashes’ into ball 3 and transfers the momentum to ball 3. This reaction continues from ball 3 to ball 4 and then when ball 4 ‘crashes’ into ball 5, ball 5 is able to swing an so we see the momentum being transferred from ball 1 to ball 5 and ball 5 begins to swing seemingly magically. If 2 balls were swung at the beginning we would see the same transfer of momentum except both ball 4 and ball 5 would swing instead of only ball 5.