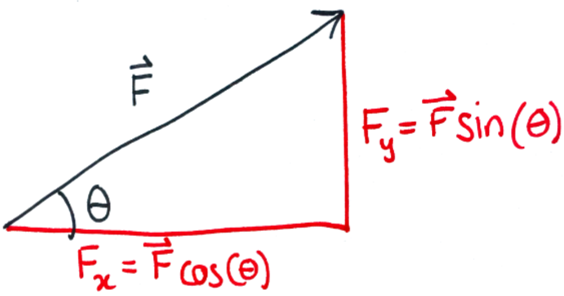
NEWTON'S THIRD LAW

$$\boldsymbol{F_x = Fcos(\theta), F_y = Fsin(\theta)}$$

CONTENT

Force, like any vector, can be broken up into its component vectors using trigonometry. Breaking a force into its components allows us to easily determine the magnitude of the force in each direction and ultimately determine the motion of the object in that direction too (which we will learn about soon).



Using trigonometry, we can create an equation for the x-component of a force and the y-component of a force. From the diagram above:

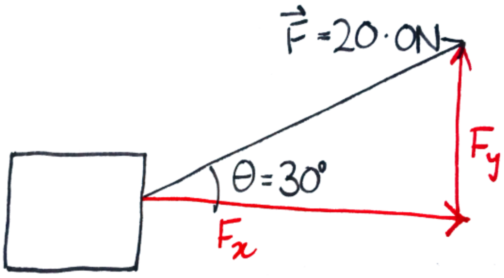
$$F_x = Fcos(\theta)$$

$$\boldsymbol{F_y = Fsin(\theta)}$$

Now instead of using trigonometry for each individual case, we can jump straight to these already derived force component equations.

EXAMPLE 1

There is a pulling force on a block that is 20.0N and at an angle of 30° to the horizontal, as in the diagram below. What are the x (horizontal) and y (vertical) components of this force?



⇒ Using the equation above for the x-component of the force we find:

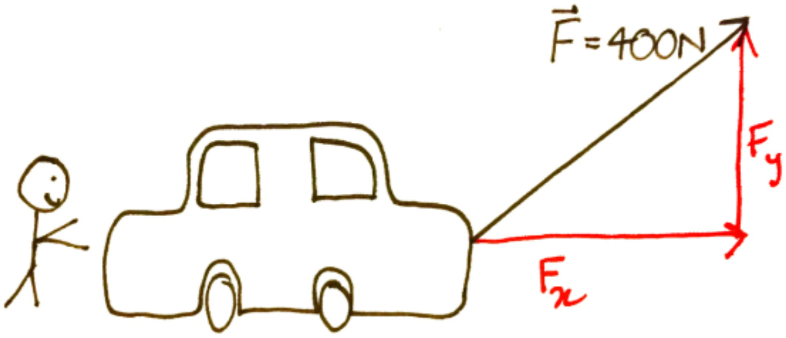
$$\begin{aligned} F_x &= \vec{F}\cos(\theta) \\ &= 20.0\cos(30) \\ &= 17.3N \end{aligned}$$

⇒ Repeating for the y-component we find:

$$F_y = \vec{F}\sin(\theta)$$
$$= 20.0\sin(30)$$
$$= \cancel{10.0N}\ 10.0N$$

Example 2

Diego's car broke down and now they need to push it to the end of the street. Diego pushes the car with a force of 400N at an angle of 45° to the horizontal. Calculate the x-component of Diego's push. Given the force required to move the car forward is at least 200N, is Diego's push large enough in the x-direction to move the car forward?



⇒ Using the equation for the x-component of the force we find:

$$F_x = \vec{F}\cos(\theta)$$
$$= 400\cos(45)$$
$$= 283N$$

⇒ And, to see if this is sufficient for the car to move forward:

$$200N < 285N$$
$$= F_x$$

⇒ Diego can push the car forward.

Investigation: Friction on Inclined Planes

INTRODUCTION

Objects on a slope tend to roll downhill due to the force of gravity. Just like objects dropped from a height, they will accelerate as they travel downwards. However, not all ramps are created equal.

Some surfaces allow objects to roll more easily than others. Consider what happens to a car left in neutral on an asphalt road versus a gravel driveway – it is very easy for the car to roll down the road, whereas on gravel it may not roll at all. Liquids like oil or water on the road can make it even easier for the car to roll

In this experiment, we will analyse the motion of objects rolling down a ramp covered with different surfaces. We will be looking at how the surface of the ramp, which has a fixed height and angle, affects the velocity of the object when it reaches the bottom. We will be looking at whether the object starts to roll and how quickly it accelerates.

1. QUESTIONING AND PREDICTING

Let us think about the aim of this investigation.

1. Which kinds of surfaces (if any) prevent the object from rolling at all?
2. Which kinds of surfaces result in the greatest acceleration?
3. For a given surface, how does the angle of the ramp affect the results?

HYPOTHESIS

Objects roll more quickly down (smooth/rough) surfaces.
Changing the surface material changes the force due to ________.

2. PLANNING INVESTIGATION

This investigation has been planned for you.

Divide up into groups. Each group will make a ramp with the same height but a different angle of inclination. Suggested materials for making the ramp are folded cardboard, or a piece of wood propped up on supports. You should collect various materials to cover the ramp: pieces of cloth, carpet samples, water, oil, sand etc.

Every group will receive a toy car to roll down the ramp. This will allow us to model the scenario described in the introduction.

Download an app on your phone which will enable you to measure the speed of the object as it leaves the ramp. Possible suggestions are; “Video Physics (Vernier)” on Apple, and “VidAnalysis Free” on Android.

1. Place the unmodified ramp on a smooth floor.
2. Hold the toy car at the top of the ramp and release it.
3. Have one person use a stopwatch to determine how long it takes for the car to reach the floor.
4. Have a second person use the app to measure the speed of the car just as it leaves the ramp.
5. Record the time and the speed for each ramp surface material.
6. Calculate the average acceleration.
7. Share results with other groups to make a table of acceleration vs. ramp angle for each surface material.

3. Conducting Investigation

Each group should fill in a table like this for their ramp.

Material	Time to base (s)	Final velocity (m/s)

Did you make any changes to the method? Did you have design problems to solve? Did you have some ‘smart’ ways of doing the investigation?

4. Processing and Analysing

To calculate the average acceleration of each object, we simply need to know the change in speed and the travel time. The initial speed is always zero, so we can simply divide the final speed by the total time.

Add a column to your previous table with the calculated accelerations.

Material	Time (s)	Final velocity (m/s)	Average acceleration (m/s^2)

Describe which surfaces result in the highest accelerations.

Describe which surfaces result in the lowest accelerations.
Are there any surfaces for which the object does not move at all?

You can now compare your results with other groups.

5. Problem solving

Gravity causes the object to accelerate downwards. The force vector points towards the centre of the Earth. We can decompose it into two components, one along the length of the ramp and one perpendicular to the ramp's surface. The component along the length of the ramp contributes to the downward motion. The steeper the ramp, the larger the magnitude of this component and hence (all other things being equal) we expect the acceleration to be greater.

However, there are other forces that can oppose this motion, decreasing the net force and hence the acceleration. One of those forces is friction.

Friction always opposes the motion of objects; if the object is travelling down the ramp, there will be a friction force directed up the ramp.

6. Conclusions

Friction always ________________ the motion of an object.

Objects roll more quickly down ________ (smooth/rough) surfaces because they provide _________ (more/less) friction.

LONGER WORKSHEET: NEWTON'S LAWS OF MOTION AND INERTIAL FRAMES OF REFERENCE

CONTENT

Before looking at Newton's first two laws of motion, we need to understand **inertial frames of reference**. An inertial frame of reference is an environment which is not accelerating. The results of Newton's Laws of Motion are only valid if they are calculated from the point of view of someone in an inertial frame of reference. The following are examples of inertial frames of reference:

- A plane at constant speed in a steady straight line, not in turbulence. This means a person on the inside does not really notice they are moving. Objects around them appear as if they are not moving and if they are dropped or thrown they behave as expected.
- A train or a bus travelling at a constant speed in a straight line on a smooth track/road without any bumps. Again, anyone on the inside of the bus or the train would not feel the motion of the bus or train if they had their eyes closed
- A person standing at rest on a train platform waiting for their train. Anything on the platform is an inertial frame of reference but anyone on a train that is slowing down to stop at the platform or speeding up as they leave the platform is not in an inertial frame of reference.

The best way to tell if a point of view is an inertial frame of reference is to imagine yourself in that situation. If you had your eyes closed, would you be able to feel if you are moving? If the answer is no, you are in an inertial frame of reference and you are not accelerating. In fact, it was Einstein who said you are unable to tell the difference between inertial frames of reference! There is no difference between a moving inertial frame of reference or a stationary one. So, any results of Newton's Laws of Motion applied in different inertial frames of reference are all still valid.

The following are examples where you cannot apply Newton's Laws of Motion because these frames of reference are accelerating.

- Passengers on a plane that is experiencing turbulence, taking off or landing. When you're on a plane in these situations you can feel the plane moving.
- A car turning around a corner. When you're in the car going around the corner you can feel your body move in the opposite direction to the turn

Even though the Earth is rotating on its axis and orbiting the Sun, this motion is small enough that we can consider someone standing on the Earth as someone in an approximate inertial frame.

EXAMPLE 1

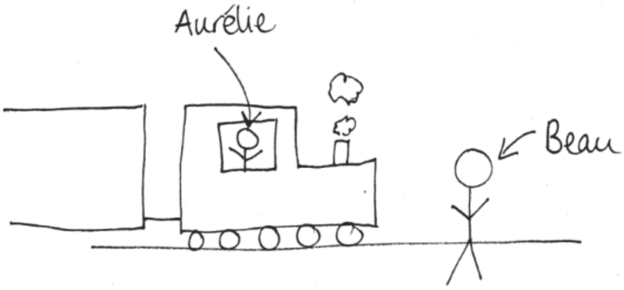
Which of the following frames of reference are inertial:

A skateboarder going down a hill getting faster and faster	Non-inertial since the skateboarder is accelerating
A train travelling at a constant speed on smooth tracks	Inertial, the train is not accelerating and not experiencing any bumps
A person sitting on a bench waiting for their bus	Inertial, the person is stationary (not moving) and so not accelerating

Example 2

Beau is standing on a train platform waiting for their train into the city. While they're waiting a train travels past at a constant velocity on smooth tracks but does not stop at Beau's platform. On the train is Aurélie. Using inertial frames of reference, discuss the differences between Beau and Aurélie's observations.

⇒ Firstly, we draw a diagram of the situation so that we can picture ourselves in each inertial frame of reference.



⇒ The first inertial frame of reference we will consider is Beau's. Since Beau is standing still on the platform, they are not accelerating and their frame of reference is inertial. As the train travels past the platform, it appears to Beau that Aurélie is moving past at a constant velocity.

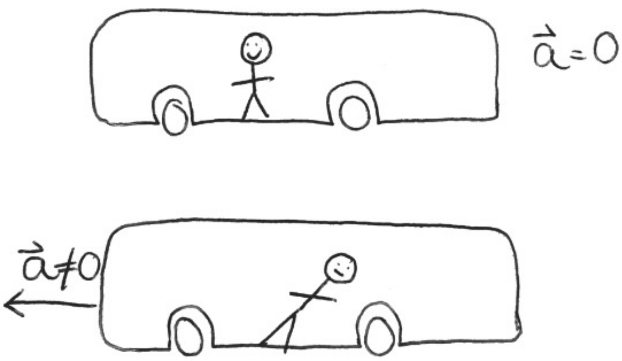
⇒ Now, if we consider Aurélie's frame of reference. Firstly, we need to check it is an inertial frame of reference. Since the train travelling with a constant velocity, on smooth tracks and in a straight line, the train and anyone inside it is in an inertial frame. So Aurélie is also in an inertial frame of reference. As the train travels past the platform, it seems to Aurélie that she is motionless while Beau and the platform and moving with a constant velocity in the opposite direction to the velocity of the train that Beau observed.

⇒ While in one frame of reference Beau appears to be the object that is motionless and Aurélie the object that's moving compared to the other frame where Aurélie appears motionless and Beau appears to be moving, both observations are valid! They seem to produce contradictory results but as both observations were made in an inertial frame of reference, both are completely valid.

Content

Newton's laws of motion physically describe the motions of objects we observe every day. His first law, the law of inertia, states:

In an inertial frame of reference, if the resultant force acting upon an object is zero, then the object will be at rest or moving with constant velocity



This means if there is no net force acting on an object, the motion will remain unchanged, all objects resist a change in their motion. For example, a hockey puck gliding along an icy (frictionless) surface will go on forever or until something stops it (like another player or the net of the goal). The motion of astronauts at the ISS is another example. The astronauts are in a permanent state of free fall which means all the forces acting on them are balanced, which means they seem to float around in the station. If an astronaut pushes off from the wall of the ISS they move away with a constant velocity (and would continue to do so) until they hit the opposite wall.
You can feel the effects of Newton's First Law when travelling on a bus that suddenly brakes. When the bus is travelling forward with constant velocity, if you close your eyes you can't tell you are moving forward (so we must be in an inertial frame of reference). However, if the bus stops, your body will resist the change in its motion and continue to move in the same direction. As a result, jerk forward and it feels like someone has pushed you forward. The same happens in the opposite direction when the bus suddenly moves forward.

Newton's Second Law of motion connects the net force acting on an object with the acceleration of the object:

If there is an unbalanced force acting, there is an acceleration in the direction of the net force

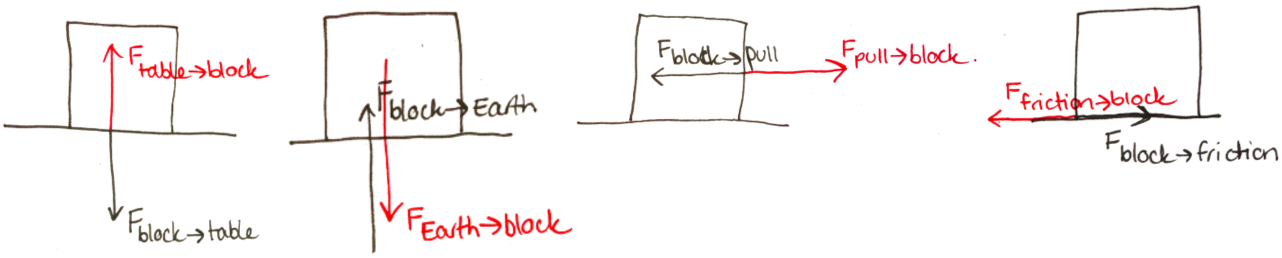
$$\vec{\boldsymbol{F}}_{\boldsymbol{net}} = \boldsymbol{m}\vec{\boldsymbol{a}}$$

Where $\vec{F}$ is the net force acting on the object, m is the mass of the object at rest and $\vec{a}$ is the acceleration of the object. This means any object that has a non-zero acceleration has a non-zero net force and vice versa and the more massive an object is, the more force is required to accelerate it. For example, throwing a netball requires less force from the player than throwing a brick.

Example 1

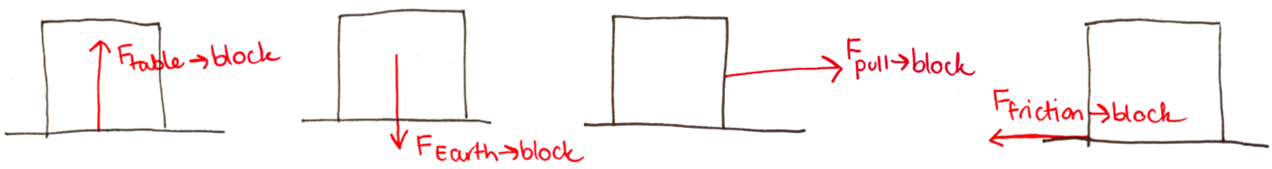
A block is being pulled across a table with a horizontal force. The force of the pull is so small the block isn't moving anywhere, it is still at rest. Draw all the forces acting on the block and qualitatively explain them.

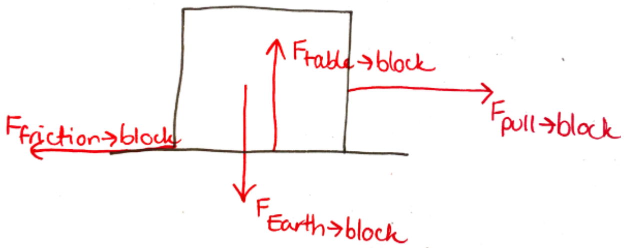
⇒ First, we shall draw a diagram of the whole scenario with all the forces present, we will then pick out the ones that are acting on the block as these are the only ones we are concerned with.



⇒ So, there is a force exerted by the table on the block, $\vec{F}_{table-block}$, the force of the block on the table, $\vec{F}_{block-table}$. The force of gravity, exerted by the Earth on the block, $\vec{F}_{Earth-block}$, and the force of gravity exerted by the block on the Earth, $\vec{F}_{block-Earth}$. The force of the pull on the block, $\vec{F}_{pull-block}$, and its reaction force of the block on the pull, $\vec{F}_{block-pull}$. Finally, the force of friction exerted by the table on the block, $\vec{F}_{friction-block}$, and the force of the friction exerted by the block on the table, $\vec{F}_{block-friction}$.

⇒ Picking out the ones that are acting on the block only to calculate our net force acting on the block we have: the contact force of the table, $\vec{F}_{table-block}$, gravity $\vec{F}_{Earth-block}$, the pulling force $\vec{F}_{pull-block}$, the friction force $\vec{F}_{friction-block}$.



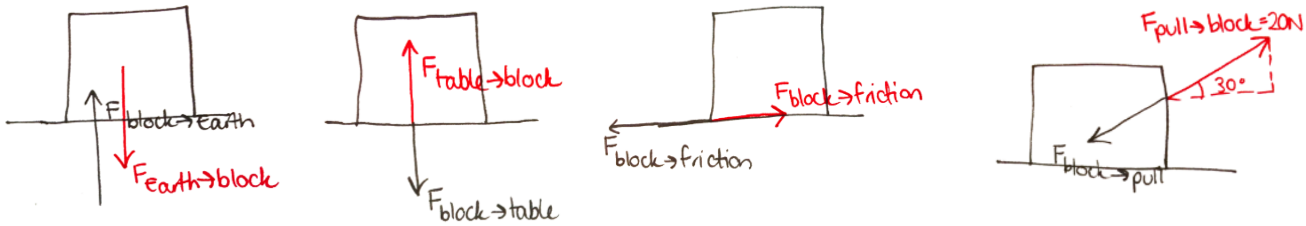


⇒ Qualitatively explaining the situation: We know from Newton's First Law that since the block is at rest, there must be a total net force of zero acting on it. So, the force of gravity (exerted by the Earth on the block) must balance the contact force of the table on the block. Similarly, the force of the pull must be balanced by the force of the friction on the block.

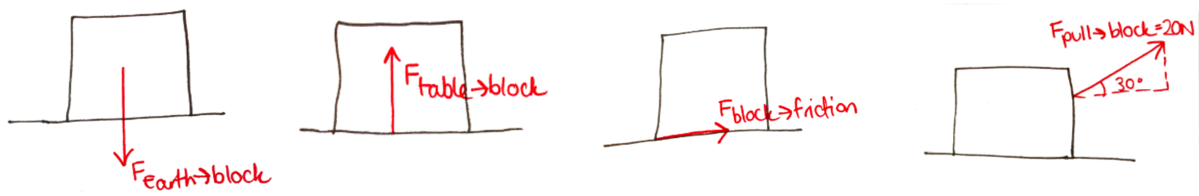
EXAMPLE 2

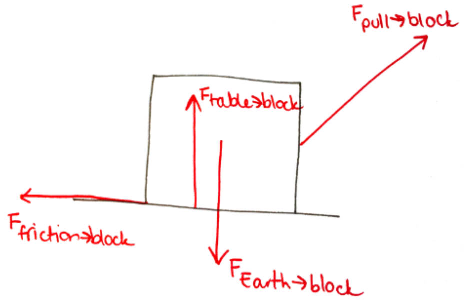
A block of mass 2kg, is being pulled along a table. The pulling force is 20N at an angle of 30° to the horizontal, as drawn below. Add in all the other forces acting on the block, assuming there is a force due to friction of 10N. What is total horizontal acceleration? Resolve the vertical forces and explain why the block doesn't fly upwards.

⇒ So, firstly we draw a diagram adding all the forces present.



We have the force due to gravity, $\vec{F}_{Earth-block}$, and its force pair the force of gravity exerted by the block on the Earth $\vec{F}_{block-Earth}$. The contact force exerted by the table on the block, $\vec{F}_{table-block}$, the contact force of the block on the table, $\vec{F}_{block-table}$, and we have the force due to friction exerted by the table on the block $\vec{F}_{friction-block} = 10N$ which has a force pair $\vec{F}_{block-table} = -10N$. And finally, the force of the pull on the block, $\vec{F}_{pull-block} = 20N$ (at 30°) and its pair the force of the block on the pull $\vec{F}_{block-pull}$. So, picking out the forces that are acting on the block we are left with: $\vec{F}_{Earth-block}$, $\vec{F}_{table-block}$ (contact force), $\vec{F}_{friction-block} = 10N$ and $\vec{F}_{pull-block} = 20N$ (at 30°).





⇒ Now, we need to resolve each of the forces into their components. Since the force due to gravity on the block and the contact force of the table on the block cancel each other out, we will ignore them in our calculations
So, we only need to calculate the vertical and horizontal component of the pulling force and the friction force on the block using trigonometry and use $\vec{F} = m\vec{a}$ to calculate the acceleration.

$$\vec{F}_x = -\vec{F}_f + \vec{F}_{P_x}$$
$$\vec{F}_{P_x} = 20\cos(30) \approx 17\text{N}$$
$$\vec{F}_f = 10\text{N}$$
$$\vec{F}_x = 17 - 10 = 7\text{N}$$

$$\vec{F}_x = m\vec{a}_x$$
$$7 = 2\vec{a}_x$$
$$\vec{a}_x = 3.5\text{ms}^{-2}$$

⇒ The horizontal acceleration is ~3.5ms^{-2}.
⇒ As for the vertical acceleration. We know the block won't fly upwards if we pull it at a slight angle, so our total vertical forces must be balanced:

$$\vec{F}_y = -\vec{F}_g + \vec{F}_{table\to block} + \vec{F}_{pull\to block}\sin(\theta)$$
$$= -\vec{F}_g + \vec{F}_{table\to block} + 20\sin(30)$$
$$= 10\text{N} - \vec{F}_g + F_{table\to block}.$$

⇒ Since the force due to gravity of the Earth on the block is a constant and won't change but we are adding the vertical force due to the pull on the block, the contact force from the table on the block must decrease to ensure everything is balanced. This means that the block won't accelerate upwards, however, it will sit on the table with less force.

$$\Rightarrow 0 = -\vec{F}_g + \vec{F}_{table\to block} + 10\text{N}$$
$$\Rightarrow \vec{F}_g = 10\text{N} + \vec{F}_{table\to block}$$

Elastic Collisions

Content

Before we look at elastic collisions, we are first going to take a step back and look at **kinetic energy** and **momentum**. The **kinetic energy** relates to the energy of motion of an object, it can be calculated using:

$$\vec{E}_K = \frac{1}{2}m\vec{v}^2$$

Where $\vec{E}_K$ is the kinetic energy, m is the mass of the object and $\vec{v}$ is the velocity of the object. So, an object that is travelling faster has more kinetic energy than an object of the same mass travelling slower.

Newton described the **momentum** of an object as a quantity of motion. It depends on the mass and motion of the object and can be calculated using the formula:

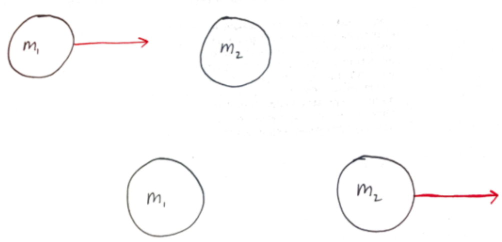
$$\vec{p} = m\vec{v}$$

When objects collide or crash into each other, we use the kinetic energy and momentum of the system before and after the crash to help understand what happened. An **elastic collision** occurs if the total kinetic energy of the system before and after the crash is the same. To calculate the total kinetic energy of the system, we add the kinetic energy of each object:

$$\sum \frac{1}{2}m\vec{v}^2$$

Where the Σ means to sum, so in this case, we add the kinetic energy for each object together.

As a simple example, consider two balls on a pool table. A red one is approaching a yellow with a constant velocity $\vec{v}$, the yellow one is stationary. The two balls collide and afterwards, the red one is stationary and the yellow one is moving away with the same velocity of the red ball before the collision. Essentially, the balls have switched velocities, so the kinetic energy of the system before the collision and after the collision is the same, making it an elastic collision.



In all collisions, the total momentum of the system is also conserved. Like with kinetic energy, to calculate the total momentum of the system we add the momentum of each object together.

$$\sum m\vec{v}$$

The total momentum for the system before the crash will *always* be the same as the total momentum after the collision.

Example

Two balls are about to have a head on collision. The first particle has a mass of 1kg travelling with a velocity of 6m/s right. The other has a mass of 0.2kg and is at rest. Their collision is perfectly elastic and the first ball is at rest after the collision while the second is travelling with an unknown velocity. What is the final velocity of the second ball?

Firstly, we begin with a diagram of the system before and after the crash.

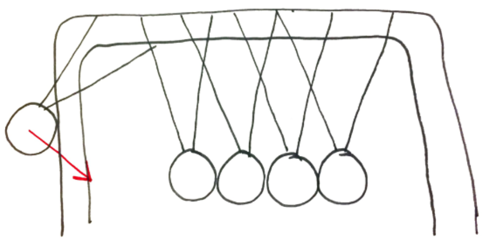
Now, we use the **conservation of momentum** and calculate the momentum before and after the crash:

$$K_{before} = K_{after}$$
$$\sum \tfrac{1}{2} m v^2 = \sum \tfrac{1}{2} m v'^2$$
$$\cancel{\tfrac{1}{2}} m_1 v_1^2 = \cancel{\tfrac{1}{2}} m_2 v_2'^2$$

$$1 \times (6)^2 = 0.2 v_2'^2$$
$$36 = 0.2 v_2'^2$$
$$180 = v_2'^2$$
$$v_2' \approx 13 ms^{-1}$$

EXAMPLE

Newton's cradle is a device with (usually) 5 balls on a held up by a wire just touching each other (see the diagram below). When one ball on the outside is raised and left to fall into the remaining 4 balls, when it crashes into them, it remains motionless while the ball on the other end starts to swing upwards and away from the other balls. This begins a chain reaction where the swinging ball is passed between the two on the end. Using the conservation of momentum and elastic collisions, explain what is happening. What would happen if two balls were initially raised and crashed into the other remaining balls?

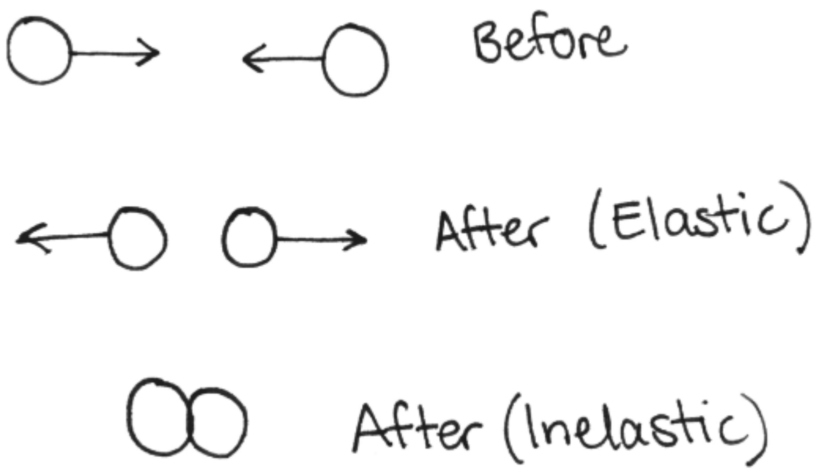


⇒ For simplicity, we shall label the balls from left to right as 1-5. So, in the diagram, ball 1 is swinging towards ball two about to crash into it. When ball 1 crashes into ball 2 it remains stationary. Due to conservation of momentum, the momentum of ball 1 must have been transferred into ball 2 (since it cannot be lost) but since ball 2 cannot swing it immediately 'crashes' into ball 3 and transfers the momentum to ball 3. This reaction continues from ball 3 to ball 4 and then when ball 4 'crashes' into ball 5, ball 5 is able to swing an so we see the momentum being transferred from ball 1 to ball 5 and ball 5 begins to swing seemingly magically. If 2 balls were swung at the beginning we would see the same transfer of momentum except both ball 4 and ball 5 would swing instead of only ball 5.

ELASTIC AND INELASTIC COLLISIONS

CONTENT

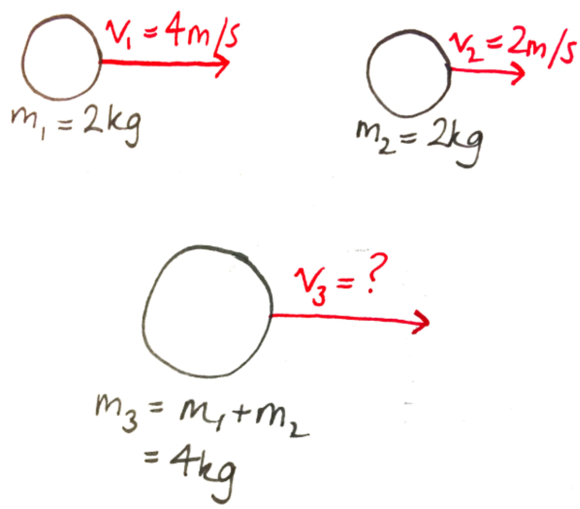
If the total kinetic energy of the system before and after a crash is the same, the crash is called an **elastic collision**. If the kinetic energy isn't the same before and after, the crash is called an **inelastic collision.** The 'missing' kinetic energy has actually been converted to other forms of energy. These include sound, heat and light. As an example of the most extreme inelastic collision, imagine two balls moving towards each other, like in the diagram below. In a perfectly elastic collision, these balls would bounce off each other and move away with the same total kinetic energy. In a perfectly inelastic collision, the two balls stick together.



In both elastic and inelastic collision, the **momentum** is the same before and after the collision.

EXAMPLE 1

A ball with a mass of 2kg is travelling with a velocity of 4m/s to the right. It is travelling towards another ball with the same mass. The second ball is travelling with a velocity of 2m/s to the right. When these two balls collide, the stick together and start travelling together. Using the conservation of momentum, calculate the velocity of the combined two balls after the collision. Is this collision elastic or inelastic?



$$
\begin{aligned}
p_{before} &= m_1\vec{v}_1 + m_2\vec{v}_2 \\
&= 2\times4 + 2\times2 \\
&= 8+4 \\
&= 12\,kgm/s
\end{aligned}
$$

⇒ We start with the conservation of momentum. First, we calculate the total momentum before the collision:

⇒ Now, we know the mass of the two balls after the collision is just the sum of the mass of the two balls. We also know, by the conservation of momentum, the momentum after the collision must be 12kgm/s. So, we solve the equation for the velocity of the two balls:

$$
\begin{aligned}
p_{after} &= 12\,kgm/s \\
&= m_3\vec{v}_3 \\
&= 4\vec{v}_3 \\
\Rightarrow 12 &= 4\vec{v}_3 \\
\Rightarrow \vec{v}_3 &= 3\,m/s
\end{aligned}
$$

⇒ Now, to determine if the collision was elastic or inelastic. To do this, we calculate the kinetic energy before and after the collision and see if they are equal.

⇒ Since the kinetic energy before and after isn't the same, the collision is inelastic.

$$
\begin{aligned}
\vec{E}_{k\text{-}before} &= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \\
&= \frac{1}{2}\times2\times(4)^2 + \frac{1}{2}\times2\times(2)^2 \\
&= 16+4 \\
&= 20\,J
\end{aligned}
$$

$$
\begin{aligned}
\vec{E}_{k\text{-}after} &= \frac{1}{2}m_3v_3^2 \\
&= \frac{1}{2}\times4\times(3)^2 \\
&= 18\,J
\end{aligned}
$$

$$\vec{E}_{k\text{-}before} \neq \vec{E}_{k\text{-}after}$$