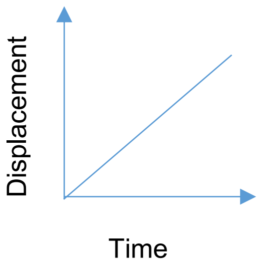
QUALITATIVE AND GRAPHICAL DESCRIPTION

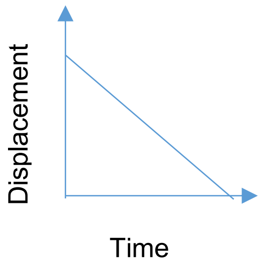
Content

We use terms like ‘speeding up’, ‘at a decreasing rate’, ‘with a steady pace’ to describe the motion of an object. To qualitatively describe the motion, we first state whether the object is moving, if their speed is increasing or decreasing and at an increasing rate or decreasing rate, then the direction of travel. For example, as a rocket launches, its speed is increasing at an increasing rate upwards.

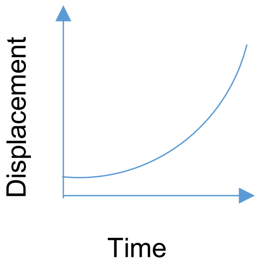
Displacement vs time graphs are also used to describe motion. The slope and shape of each of the graphs can tell us how the object is moving:



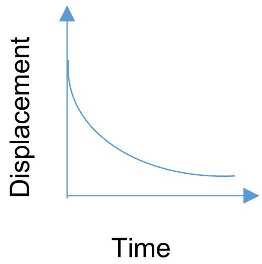
Constant positive Velocity



Constant negative velocity



Increasing positive velocity



Decreasing negative velocity

A negative slope means a negative velocity, and a curved shape means the velocity is changing (accelerating). The steeper the slope the greater the velocity.

Example

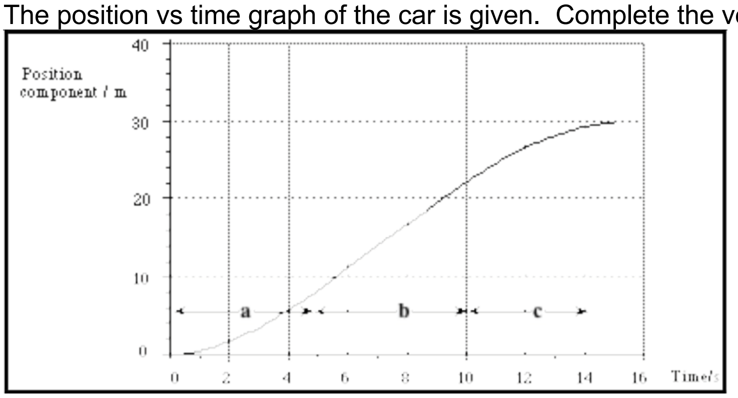
A sprinter is in a 100m race. She starts at rest and takes off when the gun fires running steadily faster until she gets to her fastest pace. Qualitatively describe the sprinters motion immediately after the gun fires.

⇒ The sprinter is moving faster and faster in the positive direction towards the finish line. Their velocity is increasing at a constant rate.

Example

1. A car is started by rolling it downhill out of gear.
2. When it is going sufficiently fast the gears are engaged. This starts the car successfully and the car is driven to the bottom of the hill
3. The car is then stopped.

The position vs time graph of the car is given. Complete the velocity vs time graph.



⇒ If we consider the velocity as the slope of the position graph in each section, 0-2s, 2-5s, 5-10s and 10-15s we can simplify the problem.

⇒ Firstly, the section 0-2s.
The position is changing with a slope that is increasing in steepness in the shape of a parabola, this means the velocity graph will appear as a straight line with a positive gradient.

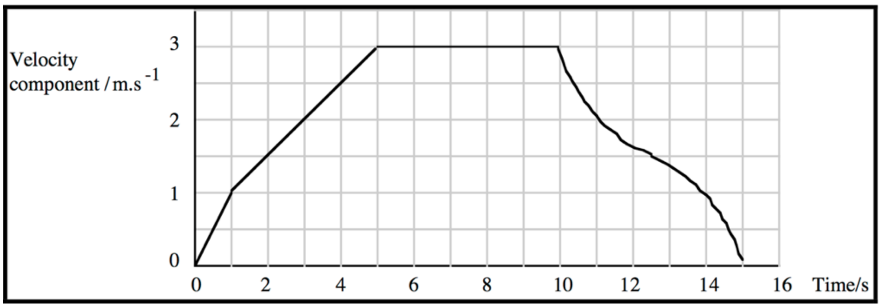
⇒ Secondly, in the section from 2-5s.

Again the position is changing with an increasing slope in the shape of a parabola. However, this time it is increasing at a slower rate. This means the velocity graph will be a straight line with a positive gradient slightly less than the gradient in the first section.

⇒ Thirdly, in the section from 5-10s
The position is changing with a constant positive slope. This means the velocity is unchanging so the velocity graph will be flat for this period

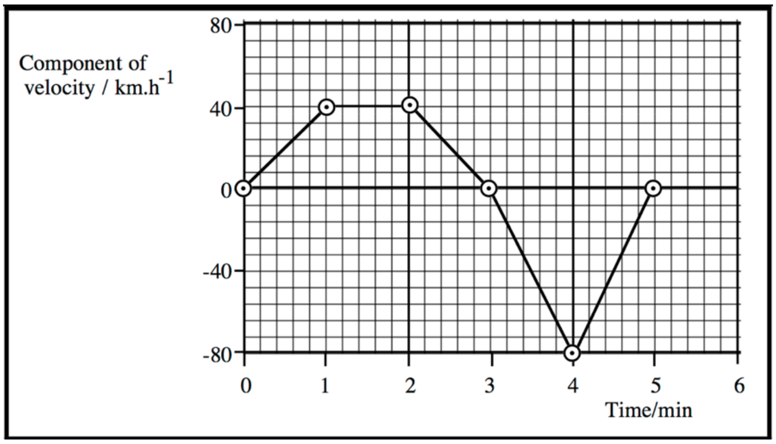
⇒ Lastly, the section from 10-15s.
The position is again changing with a positive slope but the slope is decreasing with time and levelling off until 15s where it remains constant. So the velocity is slightly more complicated, the velocity is still positive in the last section, however now it is decreasing as the car slows down to a stop at a constant position.

⇒ Combining each of the sections we obtain the final velocity graph.



EXAMPLE

Below is the velocity vs time graph of a car driving on a straight road. Qualitatively describe the motion of the car.



⇒ Again we will cut the velocity into sections and describe the motion piece by piece first.

⇒ So, firstly, considering the velocity from time=0min to time=1min. The velocity is constantly increasing in the positive direction so the car is accelerating at a constant rate forwards in the first minute.

⇒ In the second part from t=1 to t=2, the velocity is flat and positive. So the car is moving with a constant velocity in the positive direction.

⇒ From t=2min to t=3min, the velocity is still positive but is now decreasing towards zero. So the car is still moving in the positive direction however at a decreasing rate toward zero velocity which it reaches for an instant at t=3min.

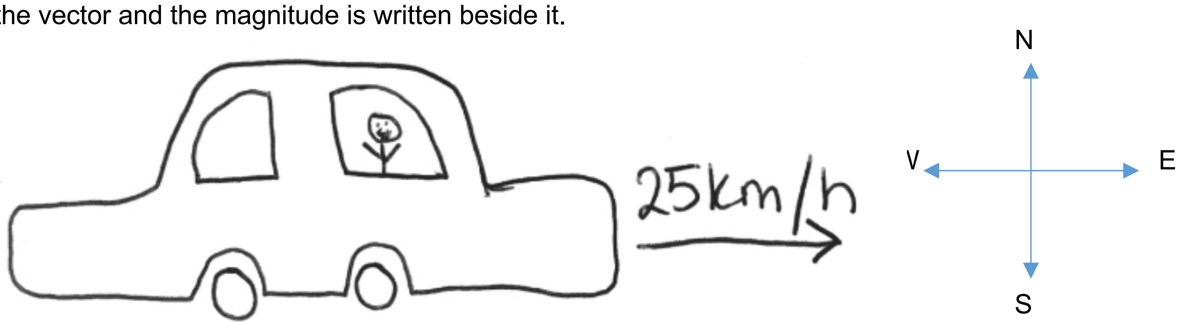
⇒ From t=3min to t=4min, the velocity is now negative and increasing in magnitude. Now the car's velocity is increasing with constant acceleration in the negative direction.

⇒ Lastly from t=4min to t=5min. The velocity is again negative, however it is approaching zero, so the magnitude is decreasing. Thus the velocity of the car is constantly decreasing in the negative direction until it reaches rest at t=5min.

VECTOR AND SCALAR QUANTITIES

CONTENT

Scalars measure only a magnitude, a number, and have no direction. For example, temperature or mass. Vectors are described by two measurements: a direction and a magnitude. Velocity is a unique example since it is the only vector where its magnitude has a name, speed. In the diagram below, the car has a velocity of 25km/h East, and a speed of 25km/h. Vectors can also be drawn using diagrams where arrows describe the direction of the vector and the magnitude is written beside it.



The direction component of vectors lets us easily add vectors together using trigonometry and Pythagoras' Theorem.
When describing the motion of an object we use both scalars and vectors. **Speed**, **distance**, d, and **time**, t, are all **scalars** since they only describe a magnitude. Whereas **displacement**, $\vec{s}$, **velocity**, $\vec{v}$, and **acceleration**, $\vec{a}$ are all vector quantities denoted with an arrow above the mathematical symbol to show a direction is included as well.

EXAMPLE

Determine which of the following quantities are vectors and which are scalars.

Quantity	Scalar or Vector	Reasoning
60km/h East	Vector	This is a velocity which has both direction and magnitude
27°C	Scalar	This is a temperature and only has magnitude
9.8m/s downwards	Vector	This is acceleration due to gravity on Earth and has both direction and magnitude
$\vec{v}$	Vector	This is the mathematical symbol for velocity with an arrow above the symbol to identify it as a vector

EXAMPLE

Pair each of the quantities with their name or description.

6000°F
16m/s
25km/h South
$\vec{s}$
34°S, 151°E
88 miles/hour

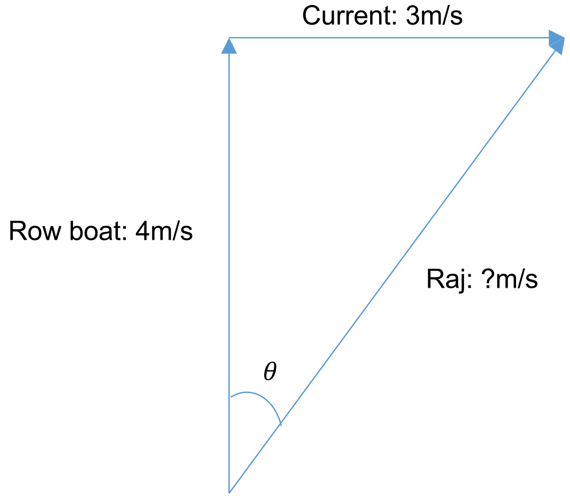
Velocity
Scalar quantity describing position
Mathematical symbol for displacement
Scalar measurement of heat
Graphical representation of velocity
Speed

It is important to note in this example, 34°S, 151°E is a scalar.
While it seems to have a direction relative to the defined zero, 34°S, 151°E is a latitude and longitude so just like (x, y) coordinates on a plane, these are coordinates for the position on the Earth.

EXAMPLE

Raj is rowing North in a river at a speed of 4m/s but there is a current flowing East at a speed of 3m/s. What is Raj's final velocity?

⇒ Firstly, we will draw a vector diagram that includes all components.



⇒ Next we will determine the speed of Raj using Pythagoras' Theorem.

$$c^2 = a^2 + b^2$$
$$c^2 = 4^2 + 3^2$$
$$c^2 = 16 + 9$$
$$c^2 = 25$$
$$c = 5$$

So Raj is travelling at 5m/s

⇒ Lastly we will determine the direction Raj is travelling using trigonometry.

$$\sin x = \frac{3}{5}$$
$$x = \sin^{-1}\frac{3}{5}$$
$$x \approx 37°$$

So Raj is travelling with a velocity of 5m/s 37° East of North.

Investigation: Falling Objects

Introduction

Falling is something that we observe all the time in everyday life, and have been learning about during our earlier classes.

We know that a force called gravity is involved. We know from experience that when things fall down they get faster and faster. But is this the case all the time?

What happens when you drop your mobile phone? What differences do you observe if you drop it onto the table, or the floor, or over a railing – which of these will cause the most damage?

In this experiment, we will analyse the motion of objects falling straight down in a straight line. This is called vertical motion in one dimension, also given the fancy name of ‘rectilinear motion’. We will be looking at their final speed when they hit the ground. We will also look at something called the average speed. Things to think about: how heavy the objects are, their masses and the height they are dropped from. Do you expect heavy objects to accelerate faster than light objects? Why/why not?

1. Questioning and predicting

So let us think about the aim of this investigation.

1. As something falls, how does its displacement changes with time?
2. What is the final velocity of the falling object?
3. How does the velocity change as the object travels?
4. What is the value of acceleration due to gravity close to Earth’s surface?

Hypothesis

As an object falls, its velocity will (increase/decrease).
Heavy objects will accelerate (faster/slower/at the same rate) as light objects.

2. Planning investigation

This investigation has been planned for you.

We will use two objects with different masses. These masses could be any shape, but if you want to ‘model’ your mobile phone, you might create an object about the same shape, size and mass of your mobile phone. Or some groups might try standard laboratory masses of 100g and 200g.

1. Place a meter ruler against the wall. This will give you a scale for measuring the displacement of the object.
2. Hold the first object at a height of 1 m above the floor.
3. Use a smartphone or digital camera to record a video of the object falling from the time when it is released.
4. The frame rate (in frames per second) of the recording is a fixed value which you can get from the camera settings. Find this value. Convert his to the time per frame. (For example, if the video is recorded at 30 fps, each frame lasts for 1/30th of a second.)
5. Look at the video frame by frame. What is the positon of your object against the ruler for each frame?
6. Record the object's position at each frame in the table on the next page. If there are a large number of frames, you may wish to take data every five or ten frames.

3. Conducting Investigation

For each object, you should take a video that clearly shows the ball falling and the scale of the ruler until the object hits the floor. Choose suitable time intervals and record its downward displacement.

Light object

Time (s)	Displacement (m)	Any other comments on motion of object and taking the video

Heavy object

Time (s)	Displacement (m)

Did you make any changes to the method? Did you have design problems to solve? Did you have some 'smart' ways of doing the investigation?

4. Processing and analysing

Plot the displacement of the ball versus time on graph paper or using a spreadsheet program.

What shape are the displacement-time graphs?

To calculate the average speed of the objects, we simply need to know the distance travelled and the amount of time it took. In the case of this experiment, the distance is always one metre.

Finding the instantaneous speed requires us to know how quickly the distance is changing at a certain time. We can do this by looking at how the distance changes between the time intervals in part 3. We can also interpolate between our data points by reading from our graph.

Plot velocity-time graphs from the data you have derived.
What shape do you expect the velocity-time plots to be?

When the data form a linear trend, we can draw a line of best fit to find the overall trend. Remember that acceleration is the change in speed versus the change in time.

Hence calculate the acceleration of the object from your velocity-time graph.

Describe the acceleration of the objects.

5. Problem solving

This is the section where we think about the results and what they mean.

When we drop an object, the force of gravity acts on it. The force on an object due to gravity is known as its weight, and can be calculated from the formula:

$$W = mg$$

The mass of the object is m and the local acceleration due to gravity is g, which is a constant.

The weight of an object depends on its mass. Does its acceleration depend on its mass?

What other factors might influence the acceleration of an object?

____

For large, light objects, the effects of wind resistance may become noticeable. What type of objects will be **least** affected by wind resistance?

An object with a constant acceleration will increase in speed by the same amount each second. Hence the velocity will increase linearly with time. (The displacement will increase with the square of the time.)

6. Conclusions

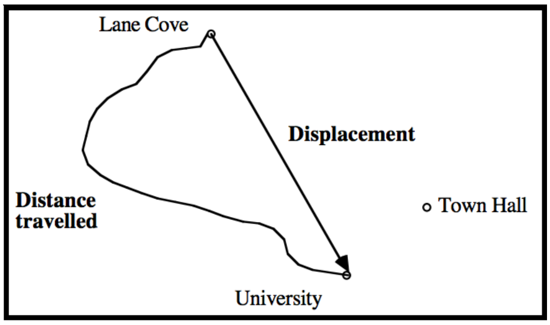
As something falls, its velocity increases ________________.

The final velocity depends on ________________.

Objects with larger masses have ________________ acceleration compared to objects with smaller masses.

EQUATIONS OF MOTION

The motion of objects can be described using both scalar and vector quantities. Scalar quantities describe a magnitude only, for example, **speed**, **distance**, d, and **time**, t, are all **scalars**. **Vectors** describe not only a magnitude but also a direction. **Displacement**, $\vec{s}$, **velocity**, $\vec{v}$, and **acceleration**, $\vec{a}$ are all vector quantities denoted with an arrow above the symbol.



The **distance travelled** is a positive number only that follows the path of the object. While, the **displacement** is the distance from the starting point to the end point and the direction of the end point relative to the start point with units of meters (m). The displacement resolved into a component is given by:

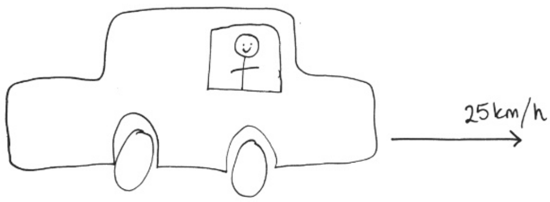
$$s = ut + \frac{1}{2}at$$

Where s is the displacement, u is the initial velocity, t is the time and a is the acceleration. **All of the vector quantities have been resolved into one component (see worksheet Vector Components).** Since it is now only in one direction and only has a magnitude, it has no arrow on top.

Velocity describes the speed <u>and</u> the direction of motion and has units of meters per second (m/s). It is best thought of as the rate of change in displacement, since displacement is also a vector. Speed on the other hand is only a magnitude and not a direction. So we consider the speed to be the rate of change in position. Velocity can be calculated using two equations:

$$v = u + at$$
$$v^2 = u^2 + 2as$$

Where v is the final velocity, u is the initial velocity and a is the acceleration.



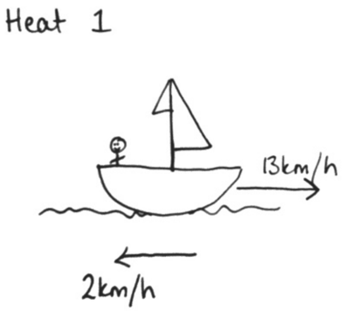
Acceleration is the magnitude of change in speed or direction and a direction of change. It has units of meters per second squared (m/s^2).

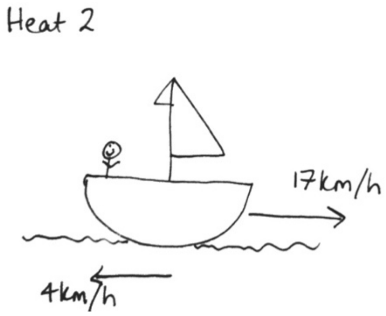
Example

Adeline competed in the world championships for rowing and wants to compare her rowing speeds between events. On one day, she was rowing forward at 13km/h but there was a current in the water flowing backwards at 2km/h. On another day, she was rowing at 17km/h but the current was 4km/h against her. Which heat was Adeline rowing faster?

$\Rightarrow$ Firstly, lets determine the differences between the two heats and draw a diagram

Heat 1	Heat 2
• Rowing velocity: 13km/h forward • Velocity of water: 2km/h backwards	• Rowing velocity: 17km/h forwards • Velocity of water: 4km/h backwards





⇒ Secondly, to calculate the final velocity of the row boat.

Heat 1:

final velocity = rowing velocity - water velocity

$$
\begin{aligned}
v_f &= v_r - v_w \\
&= 13 - 2 \\
&= 11\ \text{km/h}
\end{aligned}
$$

Heat 2:

final velocity = rowing velocity - water velocity

$$
\begin{aligned}
v_f &= v_r - v_w \\
&= 17 - 4 \\
&= 13\ \text{km/h}
\end{aligned}
$$

⇒ So, to conclude:

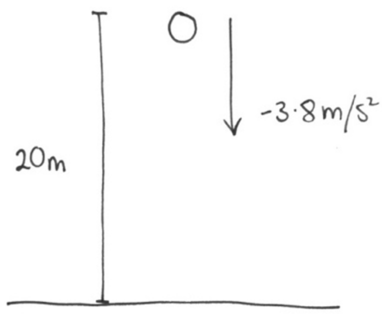
velocity heat 1 < velocity heat 2

Thus, Adeline is faster in heat 2.

Example

A rock is dropped on Mars from a height of 20 meters. What is the final velocity of the rock when it hits the ground given the acceleration due to gravity on Mars is 3.8m/s^2?

⇒ So again, first we will draw a diagram and determine the equation we need.



Initial Velocity	$u = 0$
Acceleration	$a = -3.8$
Displacement	$s = -20$
Final Velocity	$v^2 = u^2 + 2as$

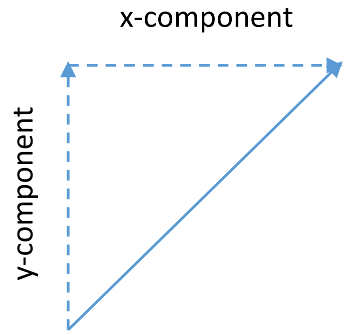
⇒ Step 2, we sub in all the values given (checking units and negatives) and solve for final velocity rounding to 2 significant figures to match the question.

$$
\begin{aligned}
v^2 &= u^2 + 2as \\
&= 0^2 + 2(-3.8 \times -20) \\
&= 152 \\
\Rightarrow v &= 12\ \text{m/s}
\end{aligned}
$$

Addition and Resolution of Vector Components

Content

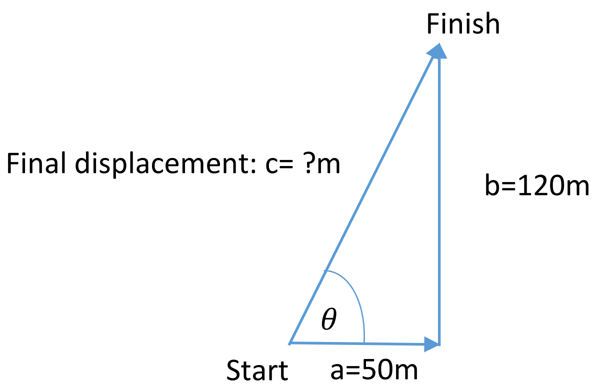
Every vector can be divided into two parts or components. We consider the vector as the hypotenuse (longest side) of a right-angle triangle. The x-component, horizontal, and the y-component, vertical, are then calculated using trigonometry. Likewise, the x-component and the y-component can be combined which is called the resultant vector.



Example

Kylie goes for a walk on a sunny day. She stats by walking East for 50m then turns left and walks North for 120m. What is her final displacement from her starting position?

⇒ To solve this, we will first draw a diagram of Kylie's path:



⇒ Now it is clear to see we only need Pythagoras' Theorem to calculate the magnitude of her displacement:

$$c^2 = a^2 + b^2$$
$$c^2 = 50^2 + 120^2$$
$$c^2 = 16{,}900$$
$$c = 130m$$

⇒ So Kylie is 130m away from where she started her walk.

Since we have her distance from the starting point we now just need to calculate θ, the angle, so we can determine her direction. We will use the $\sin(\theta)$ ratio.

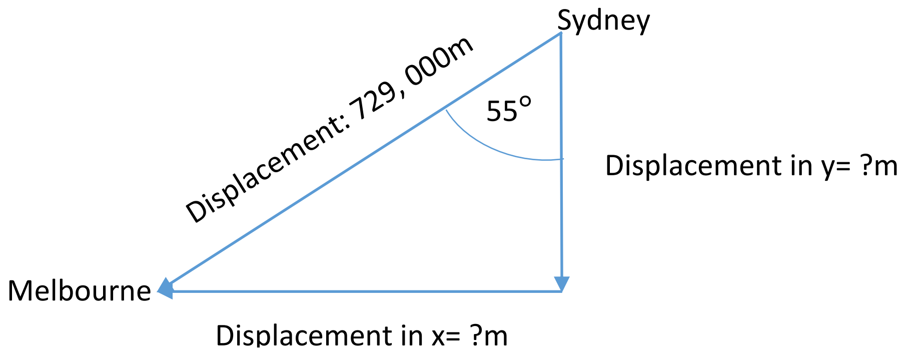
$$\sin(\theta) = \frac{b}{c}$$
$$\sin(\theta) = \frac{120}{130}$$
$$\theta = \sin^{-1}\left(\frac{120}{130}\right)$$
$$\theta = 67°$$

So Kylie is displaced 130 meters in a direction 67° East of North from where she stared.

Example

A plane flew from Sydney to Melbourne in a direct line. The distance between Sydney and Melbourne is 729km and the direction of Melbourne from Sydney is 55° West of South. What is the final displacement in x and the final displacement in y of the plane in metres?

⇒ Again, first step is to draw a diagram of the problem, converting all km to m.



⇒ Now we can use the trigonometric identities to calculate the x and y components. Lets start with the x component:

$$\sin(x) = \frac{O}{H}$$
$$\sin(55) = \frac{x}{729000}$$
$$x = 729000 \times \sin(55)$$
$$x = 5.9 \times 10^5 m$$

So the plane is roughly displaced $5.9 \times 10^5 m$ left in the x-direction, or the x-component of the displacement is $-5.9 \times 10^5 m$.

⇒ We repeat this process to calculate the y-component:

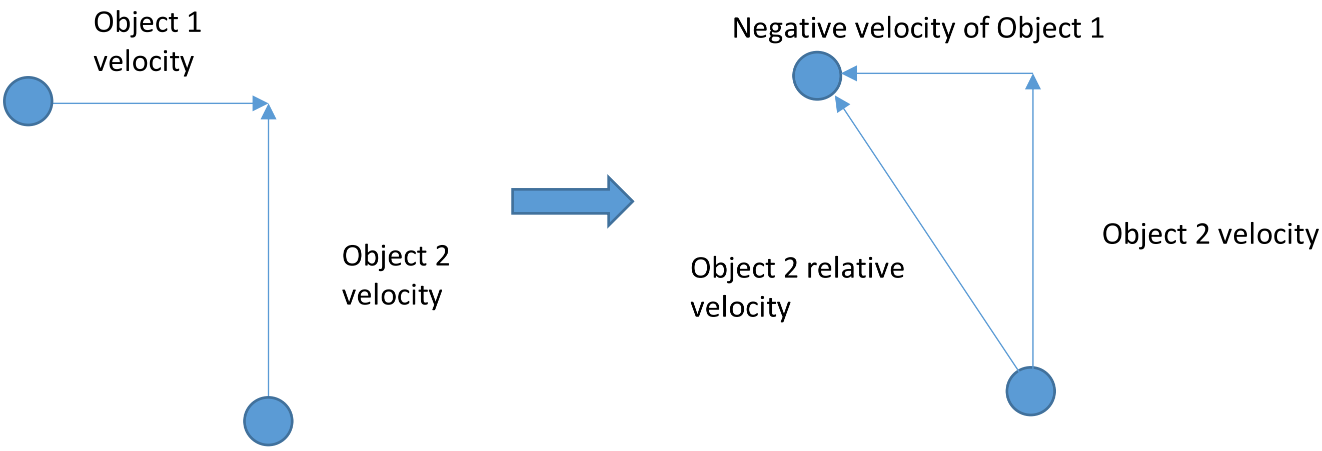
$$\cos(y) = \frac{A}{H}$$
$$\cos(55) = \frac{y}{729000}$$
$$y = 729000 \times \cos(55)$$
$$y = 418{,}137 m$$

⇒ So the plane is roughly displaced 418km downwards in the y-direction, or the y-component of the displacement is -418km.

Relative Velocity Part 1

CONTENT

Relative motion allows us to compare the motion of one object with another. First we consider one object to be at rest. Then determine the observed velocity, the relative velocity, of the other. This is calculated by adding the negative velocity of the first object to both velocities, then using vector analysis to calculate the final relative velocity.



In the diagram, we add the negative velocity of object 1 to both velocities. Now object 1 is considered at rest, and we can calculate the velocity of object 2 relative to object 1.

EXAMPLE

Two cars are driving towards each other head on. The first car is driving with a velocity of 10m/s East, the second a velocity of 15m/s West. A crash of where the relative velocity is over 100km/h is fatal for both drivers. If these cars crash, will it be fatal? Describe qualitatively the motion of the second car relative to the first.

⇒ First we will draw a diagram:

⇒ Now we will add the negative velocity of the first car to both so that car 1 is seen at rest.

⇒ Adding the velocities:

$10 - 10 = 0$ $\qquad$ $15 + 10 = 25$

⇒ So the relative velocity of the second car is 25m/s West. Qualitatively, the second car appears to be approaching the first car at a speed of 25m/s head on.

⇒ Now we need to convert m/s to km/h to see if it exceeds 100km/h.

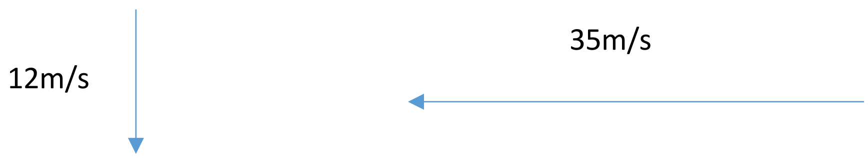
$$25ms^{-1} = 0.025kms^{-1}$$
$$0.025kms^{-1} = 1.5kmmin^{-1}$$
$$1.5kmmin^{-1} = 90kmh^{-1}$$

So the cars are travelling towards each other with a relative speed of 90km/h which is below the fatality speed.

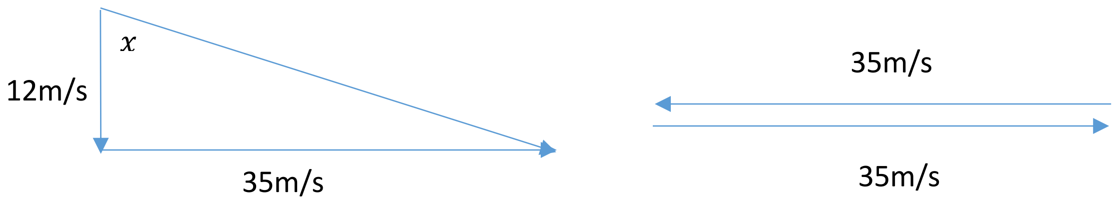
Example

A car is approaching a T intersection with a velocity of 12m/s South. A second car is approaching the intersection but with a velocity of 35m/s West. What is the velocity of the first car relative to the second?

⇒ Again, first we will draw a diagram of the problem.



⇒ Next, we add the negative velocity of the second car to both velocities:



⇒ Using Pythagoras' Theorem we calculate the relative velocity of the first car.

$$c^2 = 12^2 + 35^2$$
$$c^2 = 1369$$
$$c = 37$$

⇒ So the first car is moving with a relative speed of 37m/s now we need to calculate the direction.

$$\tan(x) = \frac{35}{12}$$
$$x = \tan^{-1}\frac{35}{12}$$
$$x = 71°$$

⇒ So the relative velocity of the first car is 37m/s 109° South East. Qualitatively, to the second car, it appears the first car is approaching with a speed of 37m/s from an angle of 19° to their right.

Relative Velocity Part 2

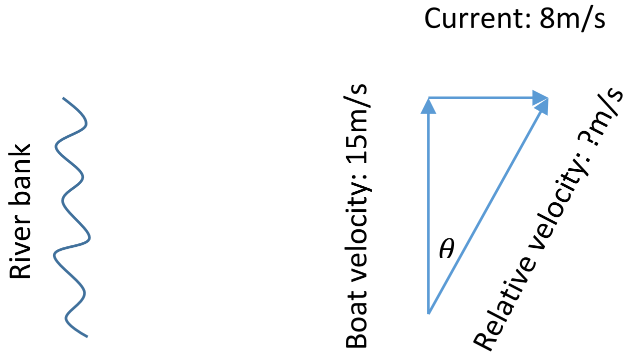
Content

Relative motion allows us to compare the motion of an object from another observer. The observer can be another moving object or a stationary reference point. We use vector analysis to add all the components of a velocity and calculate a final velocity relative to the observer

Example

A boat is flowing on a river, parallel to the river bank. The boat has a velocity of 15m/s North but there is a current flowing in the river at 8m/s East. What is the velocity of the boat relative to the river bank?

$\Rightarrow$ We begin by first drawing a diagram of the problem.



$\Rightarrow$ Using Pythagoras' Theorem, we can calculate the magnitude of the relative velocity:

$$c^2 = 8^2 + 15^2$$
$$c^2 = 289$$
$$c = 17$$

$\Rightarrow$ So the boat is moving with a speed of 37m/s relative to the bank now we need to calculate the direction.

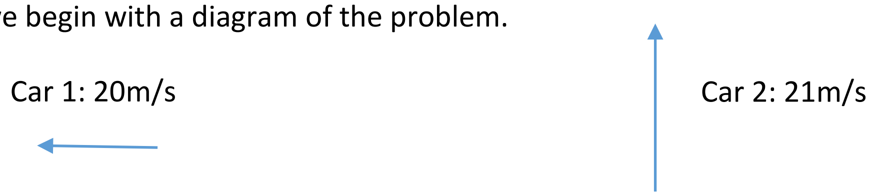
$$\tan(\theta) = \frac{8}{15}$$
$$\theta = \tan^{-1}\frac{8}{15}$$
$$\theta = 28°$$

$\Rightarrow$ So the boat is moving 17m/s 28° North East relative to the bank.

Example

Two cars are driving away from each other at right angles. The first car is driving West at a speed of 20m/s. The second car is driving North at a speed of 21m/s. What is the relative velocity of the second car? Qualitatively describe the motion of the second car as observed by the first.

$\Rightarrow$ First we begin with a diagram of the problem.



⇒ Next we add the negative velocity of car 1 to both velocities and calculate the new velocity using trigonometry.

$$20 - 20 = 0$$

$$c^2 = 21^2 + 20^2$$
$$c^2 = 841$$
$$c = 29$$

⇒ So the magnitude of the relative velocity is 29m/s. Now to calculate the direction using trigonometry.

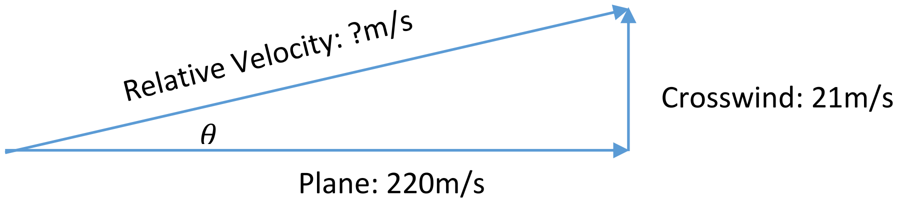
$$\tan(\theta) = \frac{20}{21}$$
$$\theta = \tan^{-1}\frac{20}{21}$$
$$\theta = 44°$$

⇒ The second car has a relative velocity of 29m/s 44° North East. Qualitatively, to the first car it appears the second car is moving away at 29m/s 44° North East.

Example

A plane is flying in a crosswind. The velocity of the plane is 220m/s East and the crosswind is 21m/s North. What is the velocity of the plane relative to the ground? Qualitatively describe what a person standing on the ground looking at the plane would see.

⇒ As always, we will begin with a diagram of the problem.



⇒ Using Pythagoras' Theorem, we calculate the magnitude of the relative velocity.

$$c^2 = 220^2 + 21^2$$
$$c^2 = 48841$$
$$c = 221$$

So the magnitude of the velocity is 221m/s, now for the direction.

$$\tan(\theta) = \frac{21}{220}$$
$$\theta = \tan^{-1}\frac{21}{220}$$
$$\theta = 5°$$

The relative velocity of the plane is 221m/s 085° North East. To an observer on the ground the plane appears to be moving 85° North East with a speed of 221m/s.