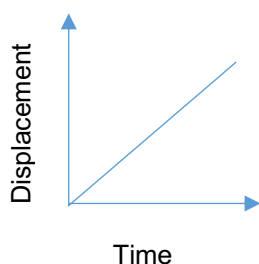


QUALITATIVE AND GRAPHICAL DESCRIPTION

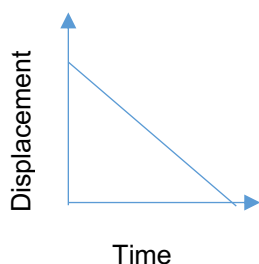
CONTENT

We use terms like 'speeding up', 'at a decreasing rate', 'with a steady pace' to describe the motion of an object. To qualitatively describe the motion, we first state whether the object is moving, if their speed is increasing or decreasing and at an increasing rate or decreasing rate, then the direction of travel. For example, as a rocket launches, its speed is increasing at an increasing rate upwards.

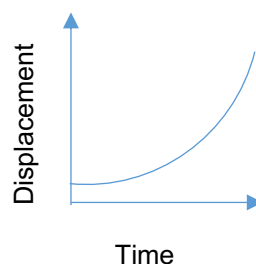
Displacement vs time graphs are also used to describe motion. The slope and shape of each of the graphs can tell us how the object is moving:



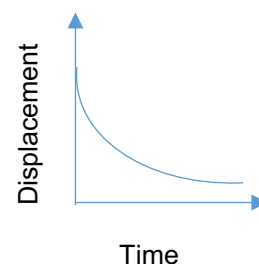
Constant positive Velocity



Constant negative velocity



Increasing positive velocity



Decreasing negative velocity

A negative slope means a negative velocity, and a curved shape means the velocity is changing (accelerating). The steeper the slope the greater the velocity.

EXAMPLE

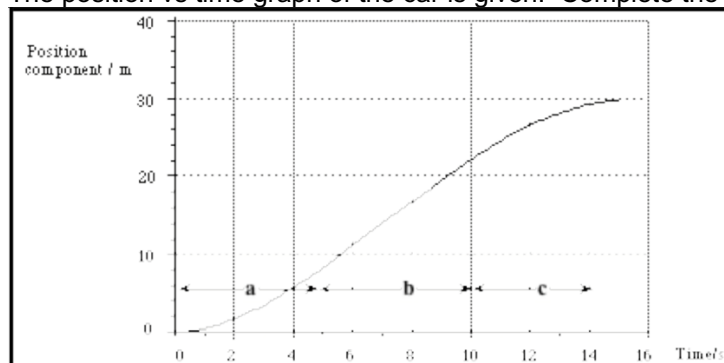
A sprinter is in a 100m race. She starts at rest and takes off when the gun fires running steadily faster until she gets to her fastest pace. Qualitatively describe the sprinters motion immediately after the gun fires.

- ⇒ The sprinter is moving faster and faster in the positive direction towards the finish line. Their velocity is increasing at a constant rate.

EXAMPLE

1. A car is started by rolling it downhill out of gear.
2. When it is going sufficiently fast the gears are engaged. This starts the car successfully and the car is driven to the bottom of the hill
3. The car is then stopped.

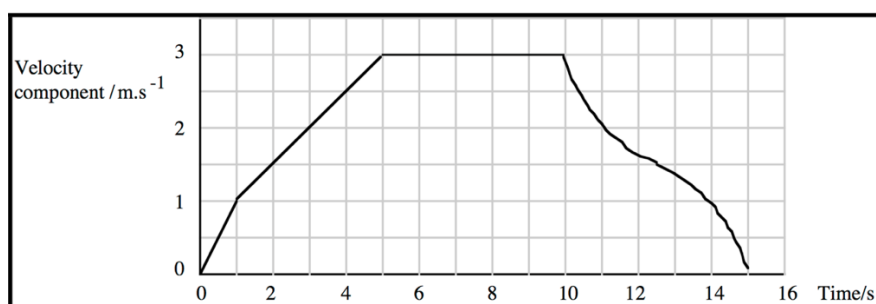
The position vs time graph of the car is given. Complete the velocity vs time graph.



- ⇒ If we consider the velocity as the slope of the position graph in each section, 0-2s, 2-5s, 5-10s and 10-15s we can simplify the problem.
- ⇒ Firstly, the section 0-2s.
The position is changing with a slope that is increasing in steepness in the shape of a parabola, this means the velocity graph will appear as a straight line with a positive gradient.
- ⇒ Secondly, in the section from 2-5s.

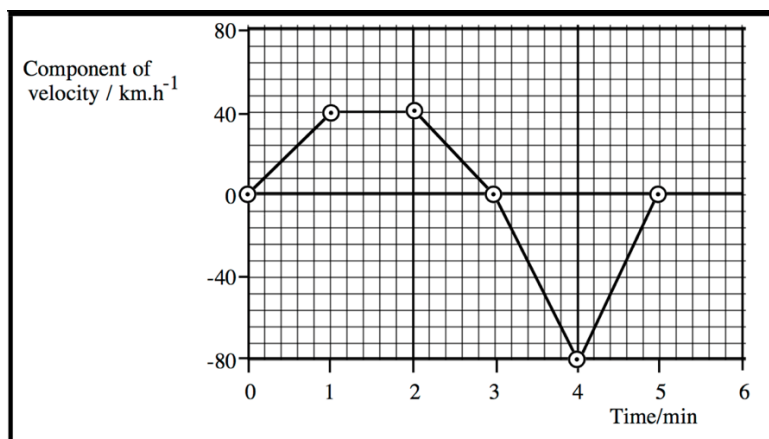
Again the position is changing with an increasing slope in the shape of a parabola. However, this time it is increasing at a slower rate. This means the velocity graph will be a straight line with a positive gradient slightly less than the gradient in the first section.

- ⇒ Thirdly, in the section from 5-10s
The position is changing with a constant positive slope. This means the velocity is unchanging so the velocity graph will be flat for this period
- ⇒ Lastly, the section from 10-15s.
The position is again changing with a positive slope but the slope is decreasing with time and levelling off until 15s where it remains constant. So the velocity is slightly more complicated, the velocity is still positive in the last section, however now it is decreasing as the car slows down to a stop at a constant position.
- ⇒ Combining each of the sections we obtain the final velocity graph.



EXAMPLE

Below is the velocity vs time graph of a car driving on a straight road. Qualitatively describe the motion of the car.



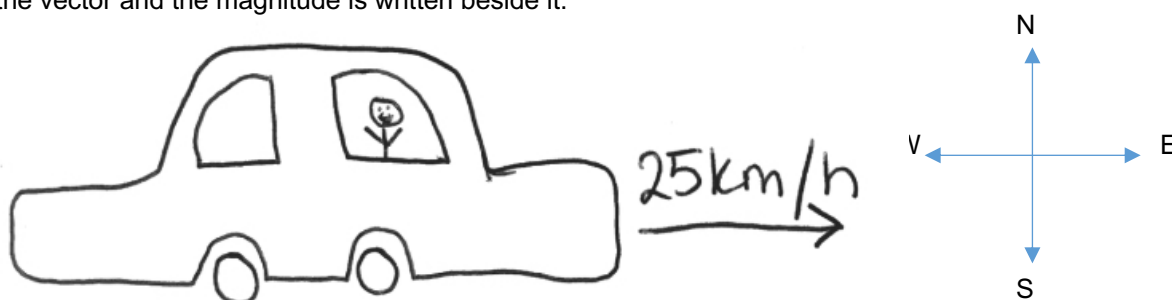
- ⇒ Again we will cut the velocity into sections and describe the motion piece by piece first.
- ⇒ So, firstly, considering the velocity from time=0min to time=1min. The velocity is constantly increasing in the positive direction so the car is accelerating at a constant rate forwards in the first minute.
- ⇒ In the second part from t=1 to t=2, the velocity is flat and positive. So the car is moving with a constant velocity in the positive direction.
- ⇒ From t=2min to t=3min, the velocity is still positive but is now decreasing towards zero. So the car is still moving in the positive direction however at a decreasing rate toward zero velocity which it reaches for an instant at t=3min.
- ⇒ From t=3min to t=4min, the velocity is now negative and increasing in magnitude. Now the car's velocity is increasing with constant acceleration in the negative direction.
- ⇒ Lastly from t=4min to t=5min. The velocity is again negative, however it is approaching zero, so the magnitude is decreasing. Thus the velocity of the car is constantly decreasing in the negative direction until it reaches rest at t=5min.



VECTOR AND SCALAR QUANTITIES

CONTENT

Scalars measure only a magnitude, a number, and have no direction. For example, temperature or mass. Vectors are described by two measurements: a direction and a magnitude. Velocity is a unique example since it is the only vector where its magnitude has a name, speed. In the diagram below, the car has a velocity of 25km/h East, and a speed of 25km/h. Vectors can also be drawn using diagrams where arrows describe the direction of the vector and the magnitude is written beside it.



The direction component of vectors lets us easily add vectors together using trigonometry and Pythagoras' Theorem.

When describing the motion of an object we use both scalars and vectors. **Speed**, **distance**, d , and **time**, t , are all **scalars** since they only describe a magnitude. Whereas **displacement**, \vec{s} , **velocity**, \vec{v} , and **acceleration**, \vec{a} are all vector quantities denoted with an arrow above the mathematical symbol to show a direction is included as well.

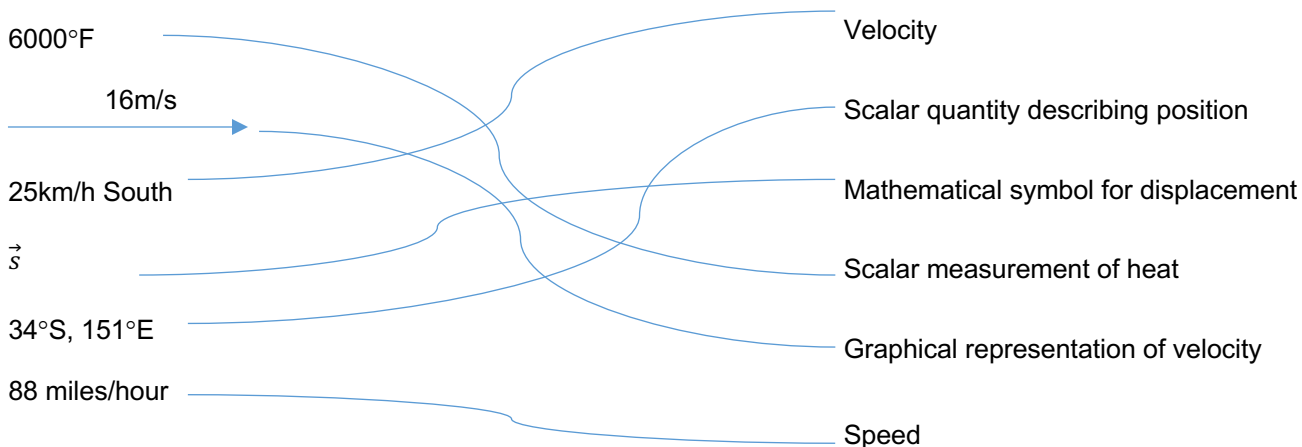
EXAMPLE

Determine which of the following quantities are vectors and which are scalars.

Quantity	Scalar or Vector	Reasoning
60km/h East	Vector	This is a velocity which has both direction and magnitude
27°C	Scalar	This is a temperature and only has magnitude
9.8m/s downwards	Vector	This is acceleration due to gravity on Earth and has both direction and magnitude
\vec{v}	Vector	This is the mathematical symbol for velocity with an arrow above the symbol to identify it as a vector

EXAMPLE

Pair each of the quantities with their name or description.



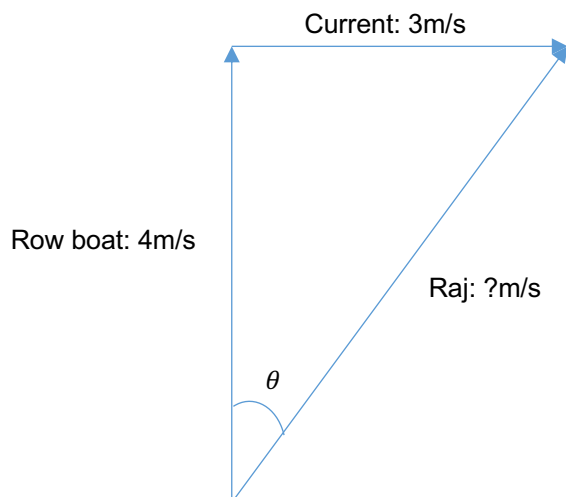
It is important to note in this example, 34°S, 151°E is a scalar.

While it seems to have a direction relative to the defined zero, 34°S, 151°E is a latitude and longitude so just like (x, y) coordinates on a plane, these are coordinates for the position on the Earth.

EXAMPLE

Raj is rowing North in a river at a speed of 4m/s but there is a current flowing East at a speed of 3m/s. What is Raj's final velocity?

⇒ Firstly, we will draw a vector diagram that includes all components.



⇒ Next we will determine the speed of Raj using Pythagoras' Theorem.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 4^2 + 3^2 \\ c^2 &= 16 + 9 \\ c^2 &= 25 \\ c &= 5 \end{aligned}$$

So Raj is travelling at 5m/s

⇒ Lastly we will determine the direction Raj is travelling using trigonometry.

$$\begin{aligned} \sin x &= \frac{3}{5} \\ x &= \sin^{-1} \frac{3}{5} \\ x &\approx 37^\circ \end{aligned}$$

So Raj is travelling with a velocity of 5m/s 37° East of North.

Investigation: Falling Objects

INTRODUCTION

Falling is something that we observe all the time in everyday life, and have been learning about during our earlier classes.

We know that a force called gravity is involved. We know from experience that when things fall down they get faster and faster. But is this the case all the time?

What happens when you drop your mobile phone? What differences do you observe if you drop it onto the table, or the floor, or over a railing – which of these will cause the most damage?

In this experiment, we will analyse the motion of objects falling straight down in a straight line. This is called vertical motion in one dimension, also given the fancy name of ‘rectilinear motion’. We will be looking at their final speed when they hit the ground. We will also look at something called the average speed. Things to think about: how heavy the objects are, their masses and the height they are dropped from. Do you expect heavy objects to accelerate faster than light objects? Why/why not?

1. QUESTIONING AND PREDICTING

So let us think about the aim of this investigation.

1. As something falls, how does its displacement changes with time?
2. What is the final velocity of the falling object?
3. How does the velocity change as the object travels?
4. What is the value of acceleration due to gravity close to Earth’s surface?

HYPOTHESIS

As an object falls, its velocity will (increase/decrease).

Heavy objects will accelerate (faster/slower/at the same rate) as light objects.

2. PLANNING INVESTIGATION

This investigation has been planned for you.

We will use two objects with different masses. These masses could be any shape, but if you want to ‘model’ your mobile phone, you might create an object about the same shape, size and mass of your mobile phone. Or some groups might try standard laboratory masses of 100g and 200g.



1. Place a meter ruler against the wall. This will give you a scale for measuring the displacement of the object.
2. Hold the first object at a height of 1 m above the floor.
3. Use a smartphone or digital camera to record a video of the object falling from the time when it is released.
4. The frame rate (in frames per second) of the recording is a fixed value which you can get from the camera settings. Find this value. Convert this to the time per frame. (For example, if the video is recorded at 30 fps, each frame lasts for $1/30^{\text{th}}$ of a second.)
5. Look at the video frame by frame. What is the position of your object against the ruler for each frame?
6. Record the object's position at each frame in the table on the next page. If there are a large number of frames, you may wish to take data every five or ten frames.

3. CONDUCTING INVESTIGATION

For each object, you should take a video that clearly shows the ball falling and the scale of the ruler until the object hits the floor. Choose suitable time intervals and record its downward displacement.

Light object

Time (s)	Displacement (m)	Any other comments on motion of object and taking the video

Heavy object

Time (s)	Displacement (m)

Did you make any changes to the method? Did you have design problems to solve? Did you have some 'smart' ways of doing the investigation?

4. PROCESSING AND ANALYSING

Plot the displacement of the ball versus time on graph paper or using a spreadsheet program.

What shape are the displacement-time graphs?

To calculate the average speed of the objects, we simply need to know the distance travelled and the amount of time it took. In the case of this experiment, the distance is always one metre.

Finding the instantaneous speed requires us to know how quickly the distance is changing at a certain time. We can do this by looking at how the distance changes between the time intervals in part 3. We can also interpolate between our data points by reading from our graph.

Plot velocity-time graphs from the data you have derived.

What shape do you expect the velocity-time plots to be?

When the data form a linear trend, we can draw a line of best fit to find the overall trend. Remember that acceleration is the change in speed versus the change in time.

Hence calculate the acceleration of the object from your velocity-time graph.

Describe the acceleration of the objects.

5. PROBLEM SOLVING

This is the section where we think about the results and what they mean.

When we drop an object, the force of gravity acts on it. The force on an object due to gravity is known as its weight, and can be calculated from the formula:

$$W = mg$$

The mass of the object is m and the local acceleration due to gravity is g , which is a constant.

The weight of an object depends on its mass. Does its acceleration depend on its mass?



What other factors might influence the acceleration of an object?

For large, light objects, the effects of wind resistance may become noticeable. What type of objects will be **least** affected by wind resistance?

An object with a constant acceleration will increase in speed by the same amount each second. Hence the velocity will increase linearly with time. (The displacement will increase with the square of the time.)

6. CONCLUSIONS

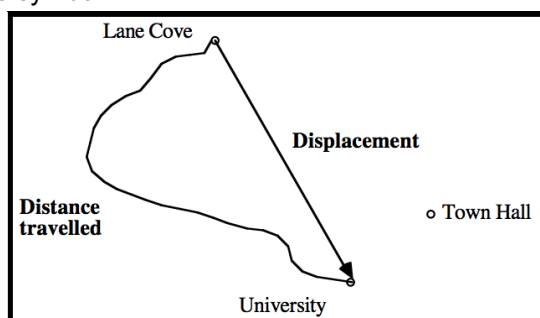
As something falls, its velocity increases _____.

The final velocity depends on _____.

Objects with larger masses have _____ acceleration compared to objects with smaller masses.

EQUATIONS OF MOTION

The motion of objects can be described using both scalar and vector quantities. Scalar quantities describe a magnitude only, for example, **speed**, **distance**, d , and **time**, t , are all **scalars**. **Vectors** describe not only a magnitude but also a direction. **Displacement**, \vec{s} , **velocity**, \vec{v} , and **acceleration**, \vec{a} are all vector quantities denoted with an arrow above the symbol.



The **distance travelled** is a positive number only that follows the path of the object. While, the **displacement** is the distance from the starting point to the end point and the direction of the end point relative to the start point with units of meters (m). The displacement resolved into a component is given by:

$$s = ut + \frac{1}{2}at^2$$

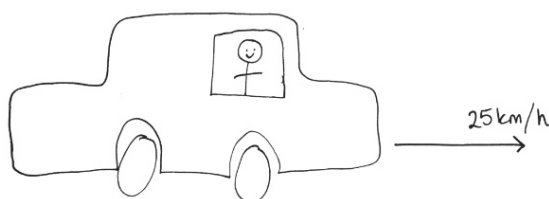
Where s is the displacement, u is the initial velocity, t is the time and a is the acceleration. **All of the vector quantities have been resolved into one component (see worksheet Vector Components)**. Since it is now only in one direction and only has a magnitude, it has no arrow on top.

Velocity describes the speed and the direction of motion and has units of meters per second (m/s). It is best thought of as the rate of change in displacement, since displacement is also a vector. Speed on the other hand is only a magnitude and not a direction. So we consider the speed to be the rate of change in position. Velocity can be calculated using two equations:

$$v = u + at$$

$$v^2 = u^2 + 2as$$

Where v is the final velocity, u is the initial velocity and a is the acceleration.



Acceleration is the magnitude of change in speed or direction and a direction of change. It has units of meters per second squared (m/s^2).

Example

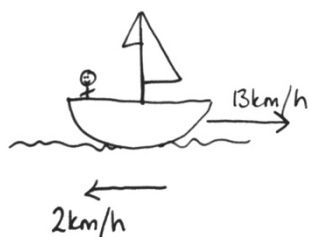
Adeline competed in the world championships for rowing and wants to compare her rowing speeds between events. On one day, she was rowing forward at 13km/h but there was a current in the water flowing backwards at 2km/h. On another day, she was rowing at 17km/h but the current was 4km/h against her. Which heat was Adeline rowing faster?

⇒ Firstly, let's determine the differences between the two heats and draw a diagram

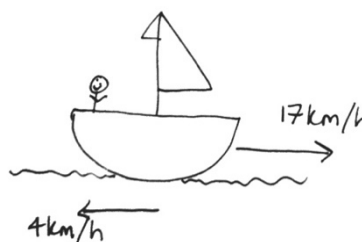
Heat 1	Heat 2
<ul style="list-style-type: none"> Rowing velocity: 13km/h forward Velocity of water: 2km/h backwards 	<ul style="list-style-type: none"> Rowing velocity: 17km/h forwards Velocity of water: 4km/h backwards



Heat 1



Heat 2



⇒ Secondly, to calculate the final velocity of the row boat.

Heat 1:

$$\begin{aligned}\text{final velocity} &= \text{rowing velocity} - \text{water velocity} \\ v_f &= v_r - v_w \\ &= 13 - 2 \\ &= 11 \text{ km/h}\end{aligned}$$

Heat 2:

$$\begin{aligned}\text{final velocity} &= \text{rowing velocity} - \text{water velocity} \\ v_f &= v_r - v_w \\ &= 17 - 4 \\ &= 13 \text{ km/h}\end{aligned}$$

⇒ So, to conclude:

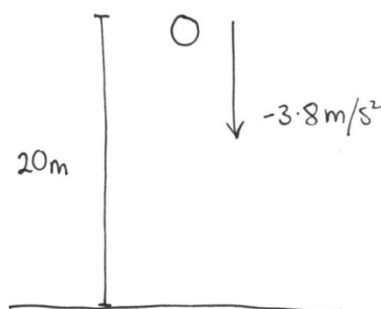
velocity heat 1 < velocity heat 2

Thus, Adeline is faster in heat 2.

Example

A rock is dropped on Mars from a height of 20 meters. What is the final velocity of the rock when it hits the ground given the acceleration due to gravity on Mars is 3.8 m/s^2 ?

⇒ So again, first we will draw a diagram and determine the equation we need.



Initial Velocity	$u = 0$
Acceleration	$a = -3.8$
Displacement	$s = -20$
Final Velocity	$v^2 = u^2 + 2as$

⇒ Step 2, we sub in all the values given (checking units and negatives) and solve for final velocity rounding to 2 significant figures to match the question.

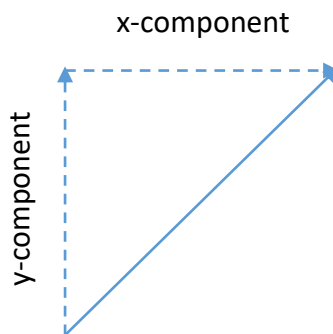
$$\begin{aligned}v^2 &= u^2 + 2as \\ &= 0^2 + 2(-3.8 \times -20) \\ &= 152 \\ \Rightarrow v &= 12 \text{ m/s}\end{aligned}$$



Addition and Resolution of Vector Components

CONTENT

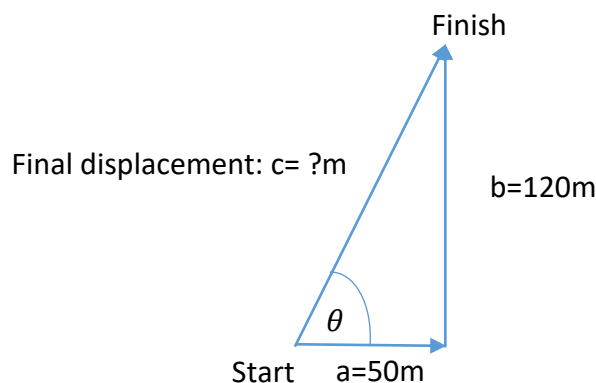
Every vector can be divided into two parts or components. We consider the vector as the hypotenuse (longest side) of a right-angle triangle. The x-component, horizontal, and the y-component, vertical, are then calculated using trigonometry. Likewise, the x-component and the y-component can be combined which is called the resultant vector.



EXAMPLE

Kylie goes for a walk on a sunny day. She starts by walking East for 50m then turns left and walks North for 120m. What is her final displacement from her starting position?

⇒ To solve this, we will first draw a diagram of Kylie's path:



⇒ Now it is clear to see we only need Pythagoras' Theorem to calculate the magnitude of her displacement:

$$\begin{aligned}c^2 &= a^2 + b^2 \\c^2 &= 50^2 + 120^2 \\c^2 &= 16,900 \\c &= 130\text{m}\end{aligned}$$

⇒ So Kylie is 130m away from where she started her walk.

Since we have her distance from the starting point we now just need to calculate θ , the angle, so we can determine her direction. We will use the $\sin(\theta)$ ratio.

$$\sin(\theta) = \frac{b}{c}$$

$$\sin(\theta) = \frac{120}{130}$$

$$\theta = \sin^{-1}\left(\frac{120}{130}\right)$$

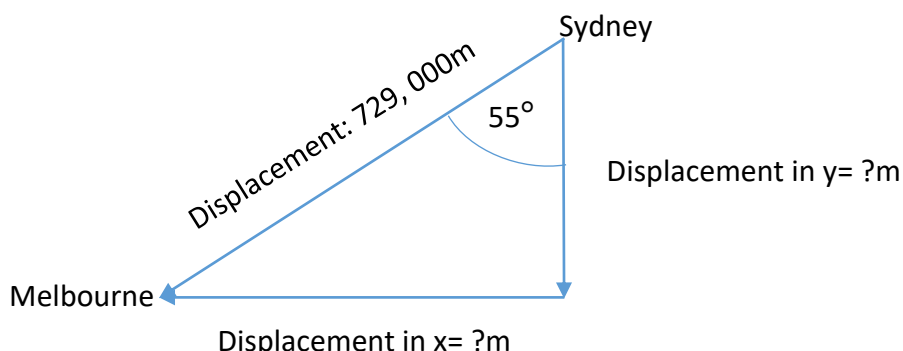
$$\theta = 67^\circ$$

So Kylie is displaced 130 meters in a direction 67° East of North from where she started.

EXAMPLE

A plane flew from Sydney to Melbourne in a direct line. The distance between Sydney and Melbourne is 729km and the direction of Melbourne from Sydney is 55° West of South. What is the final displacement in x and the final displacement in y of the plane in metres?

⇒ Again, first step is to draw a diagram of the problem, converting all km to m.



⇒ Now we can use the trigonometric identities to calculate the x and y components. Lets start with the x component:

$$\sin(x) = \frac{O}{H}$$

$$\sin(55) = \frac{x}{729000}$$

$$x = 729000 \times \sin(55)$$

$$x = 5.9 \times 10^5 m$$

So the plane is roughly displaced $5.9 \times 10^5 m$ left in the x-direction, or the x-component of the displacement is $-5.9 \times 10^5 m$.

⇒ We repeat this process to calculate the y-component:

$$\cos(y) = \frac{A}{H}$$

$$\cos(55) = \frac{y}{729000}$$

$$y = 729000 \times \cos(55)$$

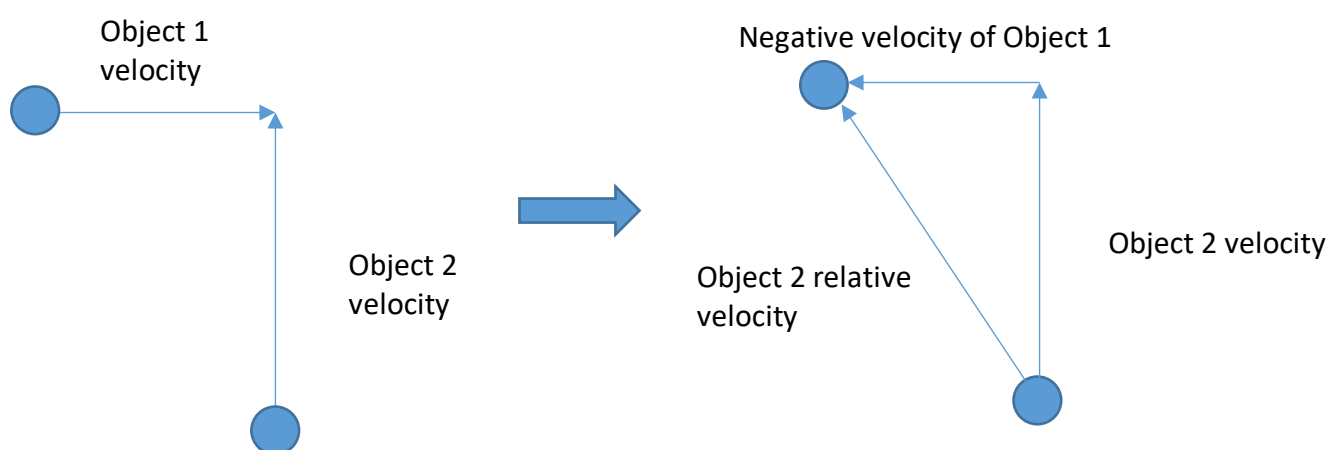
$$y = 418,137 m$$

⇒ So the plane is roughly displaced 418km downwards in the y-direction, or the y-component of the displacement is -418km.

Relative Velocity Part 1

CONTENT

Relative motion allows us to compare the motion of one object with another. First we consider one object to be at rest. Then determine the observed velocity, the relative velocity, of the other. This is calculated by adding the negative velocity of the first object to both velocities, then using vector analysis to calculate the final relative velocity.



In the diagram, we add the negative velocity of object 1 to both velocities. Now object 1 is considered at rest, and we can calculate the velocity of object 2 relative to object 1.

EXAMPLE

Two cars are driving towards each other head on. The first car is driving with a velocity of 10m/s East, the second a velocity of 15m/s West. A crash of where the relative velocity is over 100km/h is fatal for both drivers. If these cars crash, will it be fatal? Describe qualitatively the motion of the second car relative to the first.

⇒ First we will draw a diagram:



⇒ Now we will add the negative velocity of the first car to both so that car 1 is seen at rest.



⇒ Adding the velocities:

$$10 - 10 = 0$$

$$15 + 10 = 25$$

⇒ So the relative velocity of the second car is 25m/s West. Qualitatively, the second car appears to be approaching the first car at a speed of 25m/s head on.

⇒ Now we need to convert m/s to km/h to see if it exceeds 100km/h.

$$\begin{aligned} 25\text{ms}^{-1} &= 0.025\text{kms}^{-1} \\ 0.025\text{kms}^{-1} &= 1.5\text{kmmin}^{-1} \\ 1.5\text{kmmin}^{-1} &= 90\text{kmh}^{-1} \end{aligned}$$

So the cars are travelling towards each other with a relative speed of 90km/h which is below the fatality speed.

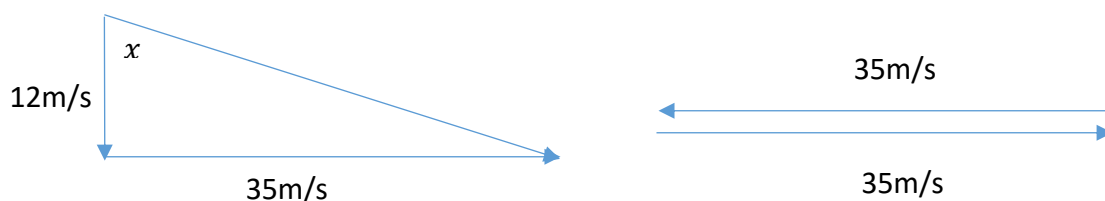
EXAMPLE

A car is approaching a T intersection with a velocity of 12m/s South. A second car is approaching the intersection but with a velocity of 35m/s West. What is the velocity of the first car relative to the second?

⇒ Again, first we will draw a diagram of the problem.



⇒ Next, we add the negative velocity of the second car to both velocities:



⇒ Using Pythagoras' Theorem we calculate the relative velocity of the first car.

$$\begin{aligned} c^2 &= 12^2 + 35^2 \\ c^2 &= 1369 \\ c &= 37 \end{aligned}$$

⇒ So the first car is moving with a relative speed of 37m/s now we need to calculate the direction.

$$\begin{aligned} \tan(x) &= \frac{35}{12} \\ x &= \tan^{-1} \frac{35}{12} \\ x &= 71^\circ \end{aligned}$$

⇒ So the relative velocity of the first car is 37m/s 109° South East. Qualitatively, to the second car, it appears the first car is approaching with a speed of 37m/s from an angle of 19° to their right.

Relative Velocity Part 2

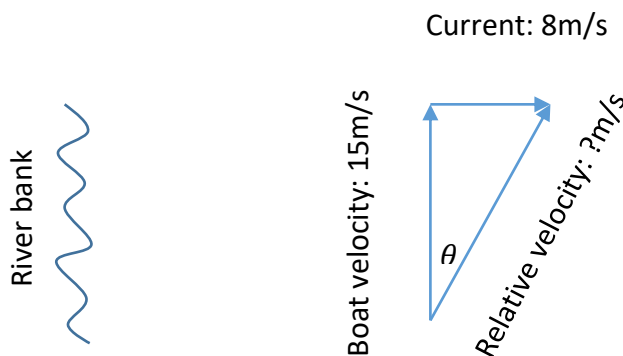
CONTENT

Relative motion allows us to compare the motion of an object from another observer. The observer can be another moving object or a stationary reference point. We use vector analysis to add all the components of a velocity and calculate a final velocity relative to the observer

EXAMPLE

A boat is flowing on a river, parallel to the river bank. The boat has a velocity of 15m/s North but there is a current flowing in the river at 8m/s East. What is the velocity of the boat relative to the river bank?

⇒ We begin by first drawing a diagram of the problem.



⇒ Using Pythagoras' Theorem, we can calculate the magnitude of the relative velocity:

$$c^2 = 8^2 + 15^2$$

$$c^2 = 289$$

$$c = 17$$

⇒ So the boat is moving with a speed of 17m/s relative to the bank now we need to calculate the direction.

$$\tan(\theta) = \frac{8}{15}$$

$$\theta = \tan^{-1} \frac{8}{15}$$

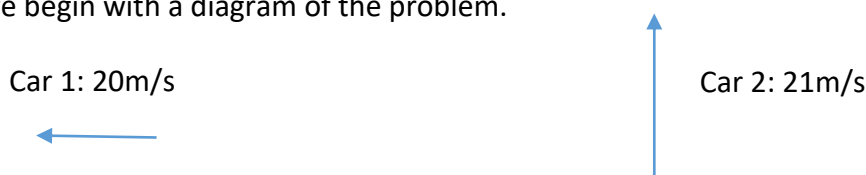
$$\theta = 28^\circ$$

⇒ So the boat is moving 17m/s 28° North East relative to the bank.

EXAMPLE

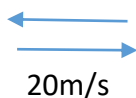
Two cars are driving away from each other at right angles. The first car is driving West at a speed of 20m/s. The second car is driving North at a speed of 21m/s. What is the relative velocity of the second car? Qualitatively describe the motion of the second car as observed by the first.

⇒ First we begin with a diagram of the problem.



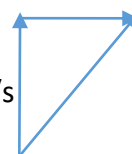
⇒ Next we add the negative velocity of car 1 to both velocities and calculate the new velocity using trigonometry.

Car 1: 20m/s



$$20 - 20 = 0$$

20m/s



Car 2: 21m/s

$$c^2 = 21^2 + 20^2$$

$$c^2 = 841$$

$$c = 29$$

⇒ So the magnitude of the relative velocity is 29m/s. Now to calculate the direction using trigonometry.

$$\tan(\theta) = \frac{20}{21}$$

$$\theta = \tan^{-1} \frac{20}{21}$$

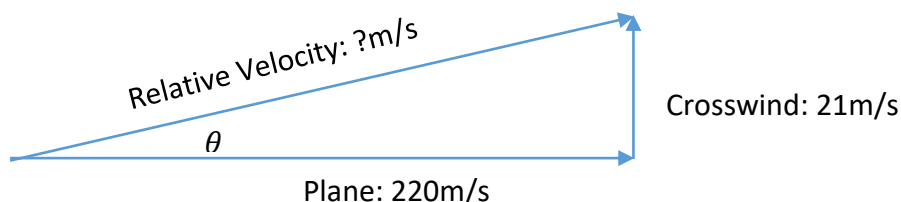
$$\theta = 44^\circ$$

⇒ The second car has a relative velocity of 29m/s 44° North East. Qualitatively, to the first car it appears the second car is moving away at 29m/s 44° North East.

EXAMPLE

A plane is flying in a crosswind. The velocity of the plane is 220m/s East and the crosswind is 21m/s North. What is the velocity of the plane relative to the ground? Qualitatively describe what a person standing on the ground looking at the plane would see.

⇒ As always, we will begin with a diagram of the problem.



⇒ Using Pythagoras' Theorem, we calculate the magnitude of the relative velocity.

$$c^2 = 220^2 + 21^2$$

$$c^2 = 48841$$

$$c = 221$$

So the magnitude of the velocity is 221m/s, now for the direction.

$$\tan(\theta) = \frac{21}{220}$$

$$\theta = \tan^{-1} \frac{21}{220}$$

$$\theta = 5^\circ$$

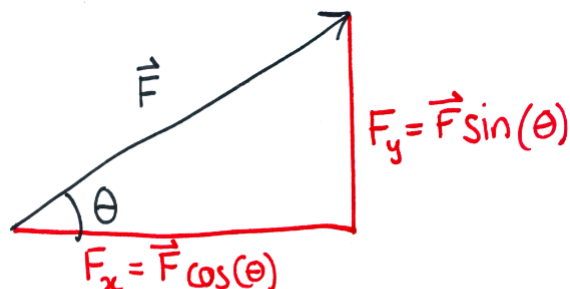
The relative velocity of the plane is 221m/s 85° North East. To an observer on the ground the plane appears to be moving 85° North East with a speed of 221m/s.

NEWTON'S THIRD LAW

$$F_x = F \cos(\theta), F_y = F \sin(\theta)$$

CONTENT

Force, like any vector, can be broken up into its component vectors using trigonometry. Breaking a force into its components allows us to easily determine the magnitude of the force in each direction and ultimately determine the motion of the object in that direction too (which we will learn about soon).



Using trigonometry, we can create an equation for the x-component of a force and the y-component of a force. From the diagram above:

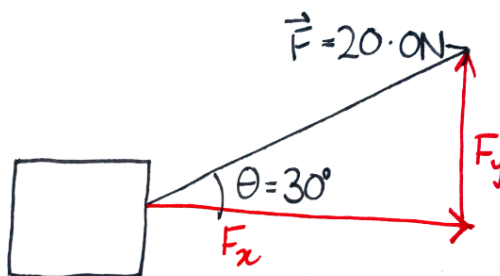
$$F_x = F \cos(\theta)$$

$$F_y = F \sin(\theta)$$

Now instead of using trigonometry for each individual case, we can jump straight to these already derived force component equations.

EXAMPLE 1

There is a pulling force on a block that is 20.0N and at an angle of 30° to the horizontal, as in the diagram below. What are the x (horizontal) and y (vertical) components of this force?



⇒ Using the equation above for the x-component of the force we find:

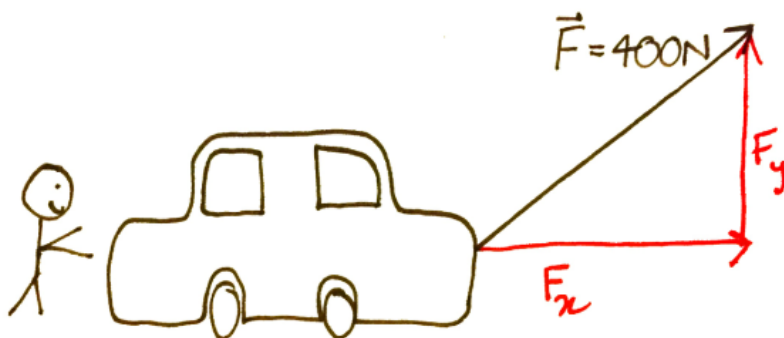
$$\begin{aligned} F_x &= F \cos(\theta) \\ &= 20.0 \cos(30) \\ &= 17.3 \text{ N} \end{aligned}$$

⇒ Repeating for the y-component we find:

$$\begin{aligned} F_y &= \vec{F} \sin(\theta) \\ &= 20.0 \sin(30) \\ &= ~~10.0~~ 10.0 \text{ N} \end{aligned}$$

EXAMPLE 2

Diego's car broke down and now they need to push it to the end of the street. Diego pushes the car with a force of 400N at an angle of 45° to the horizontal. Calculate the x-component of Diego's push. Given the force required to move the car forward is at least 200N, is Diego's push large enough in the x-direction to move the car forward?



⇒ Using the equation for the x-component of the force we find:

$$\begin{aligned} F_x &= \vec{F} \cos(\theta) \\ &= 400 \cos(45) \\ &= 283 \text{ N} \end{aligned}$$

⇒ And, to see if this is sufficient for the car to move forward:

$$\begin{aligned} 200 \text{ N} &< 285 \text{ N} \\ &= F_x \\ \Rightarrow \text{Diego can push} \\ &\text{the car forward.} \end{aligned}$$

Investigation: Friction on Inclined Planes

INTRODUCTION

Objects on a slope tend to roll downhill due to the force of gravity. Just like objects dropped from a height, they will accelerate as they travel downwards. However, not all ramps are created equal.

Some surfaces allow objects to roll more easily than others. Consider what happens to a car left in neutral on an asphalt road versus a gravel driveway – it is very easy for the car to roll down the road, whereas on gravel it may not roll at all. Liquids like oil or water on the road can make it even easier for the car to roll

In this experiment, we will analyse the motion of objects rolling down a ramp covered with different surfaces. We will be looking at how the surface of the ramp, which has a fixed height and angle, affects the velocity of the object when it reaches the bottom. We will be looking at whether the object starts to roll and how quickly it accelerates.

1. QUESTIONING AND PREDICTING

Let us think about the aim of this investigation.

1. Which kinds of surfaces (if any) prevent the object from rolling at all?
2. Which kinds of surfaces result in the greatest acceleration?
3. For a given surface, how does the angle of the ramp affect the results?

HYPOTHESIS

Objects roll more quickly down (smooth/rough) surfaces.

Changing the surface material changes the force due to _____.

2. PLANNING INVESTIGATION

This investigation has been planned for you.

Divide up into groups. Each group will make a ramp with the same height but a different angle of inclination. Suggested materials for making the ramp are folded cardboard, or a piece of wood propped up on supports. You should collect various materials to cover the ramp: pieces of cloth, carpet samples, water, oil, sand etc.

Every group will receive a toy car to roll down the ramp. This will allow us to model the scenario described in the introduction.

Download an app on your phone which will enable you to measure the speed of the object as it leaves the ramp. Possible suggestions are; “Video Physics (Vernier)” on Apple, and “VidAnalysis Free” on Android.

1. Place the unmodified ramp on a smooth floor.
2. Hold the toy car at the top of the ramp and release it.
3. Have one person use a stopwatch to determine how long it takes for the car to reach the floor.
4. Have a second person use the app to measure the speed of the car just as it leaves the ramp.
5. Record the time and the speed for each ramp surface material.
6. Calculate the average acceleration.
7. Share results with other groups to make a table of acceleration vs. ramp angle for each surface material.

3. CONDUCTING INVESTIGATION

Each group should fill in a table like this for their ramp.

Material	Time to base (s)	Final velocity (m/s)

Did you make any changes to the method? Did you have design problems to solve? Did you have some ‘smart’ ways of doing the investigation?

4. PROCESSING AND ANALYSING

To calculate the average acceleration of each object, we simply need to know the change in speed and the travel time. The initial speed is always zero, so we can simply divide the final speed by the total time.

Add a column to your previous table with the calculated accelerations.

Material	Time (s)	Final velocity (m/s)	Average acceleration (m/s^2)

Describe which surfaces result in the highest accelerations.

Describe which surfaces result in the lowest accelerations.

Are there any surfaces for which the object does not move at all?

You can now compare your results with other groups.

5. PROBLEM SOLVING

Gravity causes the object to accelerate downwards. The force vector points towards the centre of the Earth. We can decompose it into two components, one along the length of the ramp and one perpendicular to the ramp's surface. The component along the length of the ramp contributes to the downward motion. The steeper the ramp, the larger the magnitude of this component and hence (all other things being equal) we expect the acceleration to be greater.

However, there are other forces that can oppose this motion, decreasing the net force and hence the acceleration. One of those forces is friction.

Friction always opposes the motion of objects; if the object is travelling down the ramp, there will be a friction force directed up the ramp.

6. CONCLUSIONS

Friction always _____ the motion of an object.

Objects roll more quickly down _____ (smooth/rough) surfaces because they provide _____ (more/less) friction.

LONGER WORKSHEET: NEWTON'S LAWS OF MOTION AND INERTIAL FRAMES OF REFERENCE

CONTENT

Before looking at Newton's first two laws of motion, we need to understand **inertial frames of reference**. An inertial frame of reference is an environment which is not accelerating. The results of Newton's Laws of Motion are only valid if they are calculated from the point of view of someone in an inertial frame of reference. The following are examples of inertial frames of reference:

- A plane at constant speed in a steady straight line, not in turbulence. This means a person on the inside does not really notice they are moving. Objects around them appear as if they are not moving and if they are dropped or thrown they behave as expected.
- A train or a bus travelling at a constant speed in a straight line on a smooth track/road without any bumps. Again, anyone on the inside of the bus or the train would not feel the motion of the bus or train if they had their eyes closed
- A person standing at rest on a train platform waiting for their train. Anything on the platform is an inertial frame of reference but anyone on a train that is slowing down to stop at the platform or speeding up as they leave the platform is not in an inertial frame of reference.

The best way to tell if a point of view is an inertial frame of reference is to imagine yourself in that situation. If you had your eyes closed, would you be able to feel if you are moving? If the answer is no, you are in an inertial frame of reference and you are not accelerating. In fact, it was Einstein who said you are unable to tell the difference between inertial frames of reference! There is no difference between a moving inertial frame of reference or a stationary one. So, any results of Newton's Laws of Motion applied in different inertial frames of reference are all still valid.

The following are examples where you cannot apply Newton's Laws of Motion because these frames of reference are accelerating.

- Passengers on a plane that is experiencing turbulence, taking off or landing. When you're on a plane in these situations you can feel the plane moving.
- A car turning around a corner. When you're in the car going around the corner you can feel your body move in the opposite direction to the turn

Even though the Earth is rotating on its axis and orbiting the Sun, this motion is small enough that we can consider someone standing on the Earth as someone in an approximate inertial frame.

EXAMPLE 1

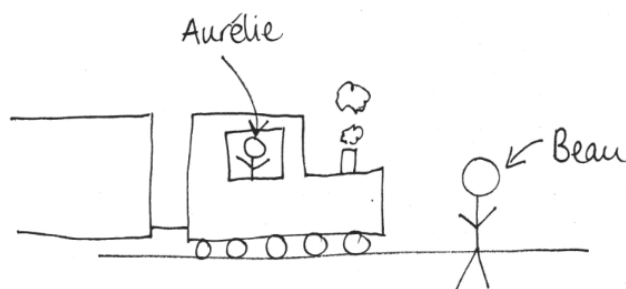
Which of the following frames of reference are inertial:

A skateboarder going down a hill getting faster and faster	Non-inertial since the skateboarder is accelerating
A train travelling at a constant speed on smooth tracks	Inertial, the train is not accelerating and not experiencing any bumps
A person sitting on a bench waiting for their bus	Inertial, the person is stationary (not moving) and so not accelerating

EXAMPLE 2

Beau is standing on a train platform waiting for their train into the city. While they're waiting a train travels past at a constant velocity on smooth tracks but does not stop at Beau's platform. On the train is Aurélie. Using inertial frames of reference, discuss the differences between Beau and Aurélie's observations.

- ⇒ Firstly, we draw a diagram of the situation so that we can picture ourselves in each inertial frame of reference.

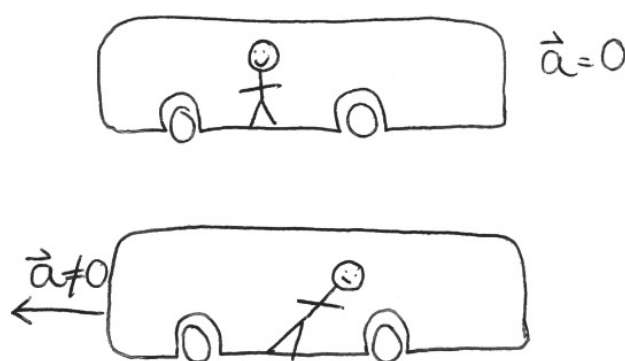


- ⇒ The first inertial frame of reference we will consider is Beau's. Since Beau is standing still on the platform, they are not accelerating and their frame of reference is inertial. As the train travels past the platform, it appears to Beau that Aurélie is moving past at a constant velocity.
- ⇒ Now, if we consider Aurélie's frame of reference. Firstly, we need to check it is an inertial frame of reference. Since the train travelling with a constant velocity, on smooth tracks and in a straight line, the train and anyone inside it is in an inertial frame. So Aurélie is also in an inertial frame of reference. As the train travels past the platform, it seems to Aurélie that she is motionless while Beau and the platform are moving with a constant velocity in the opposite direction to the velocity of the train that Beau observed.
- ⇒ While in one frame of reference Beau appears to be the object that is motionless and Aurélie the object that's moving compared to the other frame where Aurélie appears motionless and Beau appears to be moving, both observations are valid! They seem to produce contradictory results but as both observations were made in an inertial frame of reference, both are completely valid.

CONTENT

Newton's laws of motion physically describe the motions of objects we observe every day. His first law, the law of inertia, states:

In an inertial frame of reference, if the resultant force acting upon an object is zero, then the object will be at rest or moving with constant velocity



This means if there is no net force acting on an object, the motion will remain unchanged, all objects resist a change in their motion. For example, a hockey puck gliding along an icy (frictionless) surface will go on forever or until something stops it (like another player or the net of the goal). The motion of astronauts at the ISS is another example. The astronauts are in a permanent state of free fall which means all the forces acting on them are balanced, which means they seem to float around in the station. If an astronaut pushes off from the wall of the ISS they move away with a constant velocity (and would continue to do so) until they hit the opposite wall.

You can feel the effects of Newton's First Law when travelling on a bus that suddenly brakes. When the bus is travelling forward with constant velocity, if you close your eyes you can't tell you are moving forward (so we must be in an inertial frame of reference). However, if the bus stops, your body will resist the change in its motion and continue to move in the same direction. As a result, jerk forward and it feels like someone has pushed you forward. The same happens in the opposite direction when the bus suddenly moves forward.

Newton's Second Law of motion connects the net force acting on an object with the acceleration of the object:

If there is an unbalanced force acting, there is an acceleration in the direction of the net force

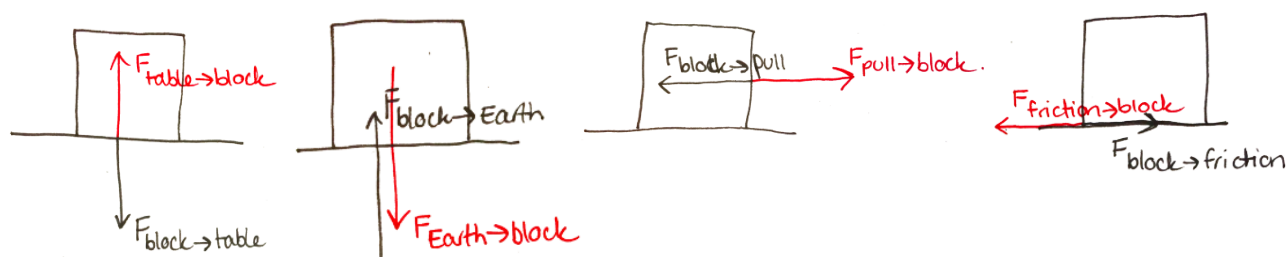
$$\vec{F}_{net} = m\vec{a}$$

Where \vec{F} is the net force acting on the object, m is the mass of the object at rest and \vec{a} is the acceleration of the object. This means any object that has a non-zero acceleration has a non-zero net force and vice versa and the more massive an object is, the more force is required to accelerate it. For example, throwing a netball requires less force from the player than throwing a brick.

EXAMPLE 1

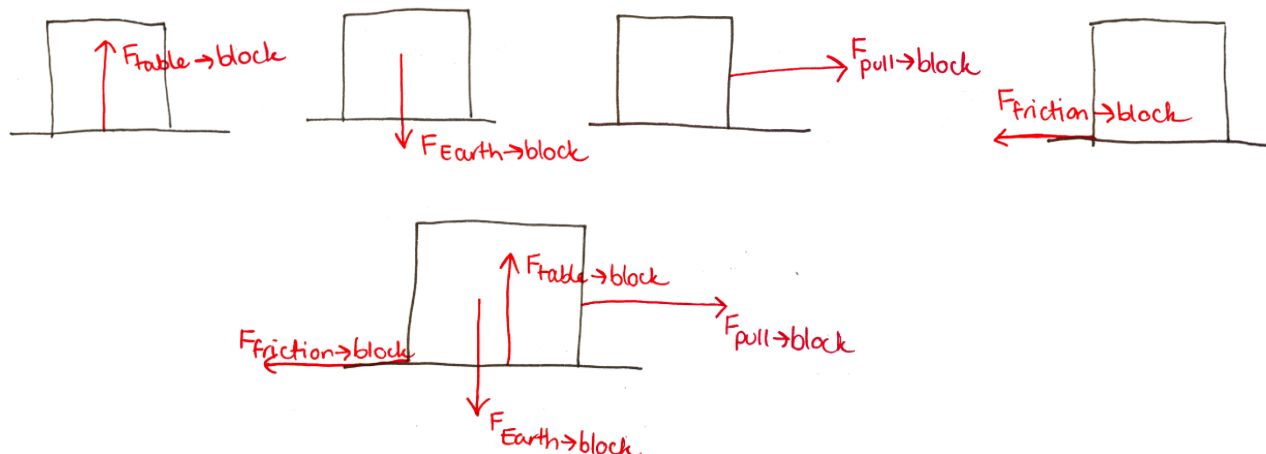
A block is being pulled across a table with a horizontal force. The force of the pull is so small the block isn't moving anywhere, it is still at rest. Draw all the forces acting on the block and qualitatively explain them.

⇒ First, we shall draw a diagram of the whole scenario with all the forces present, we will then pick out the ones that are acting on the block as these are the only ones we are concerned with.



⇒ So, there is a force exerted by the table on the block, $\vec{F}_{table \rightarrow block}$, the force of the block on the table, $\vec{F}_{block \rightarrow table}$. The force of gravity, exerted by the Earth on the block, $\vec{F}_{Earth \rightarrow block}$, and the force of gravity exerted by the block on the Earth, $\vec{F}_{block \rightarrow Earth}$. The force of the pull on the block, $\vec{F}_{pull \rightarrow block}$, and its reaction force of the block on the pull, $\vec{F}_{block \rightarrow pull}$. Finally, the force of friction exerted by the table on the block, $\vec{F}_{friction \rightarrow block}$, and the force of the friction exerted by the block on the table, $\vec{F}_{block \rightarrow friction}$.

- ⇒ Picking out the ones that are acting on the block only to calculate our net force acting on the block we have: the contact force of the table, $\vec{F}_{\text{table} \rightarrow \text{block}}$, gravity $\vec{F}_{\text{Earth} \rightarrow \text{block}}$, the pulling force $\vec{F}_{\text{pull} \rightarrow \text{block}}$, the friction force $\vec{F}_{\text{friction} \rightarrow \text{block}}$.

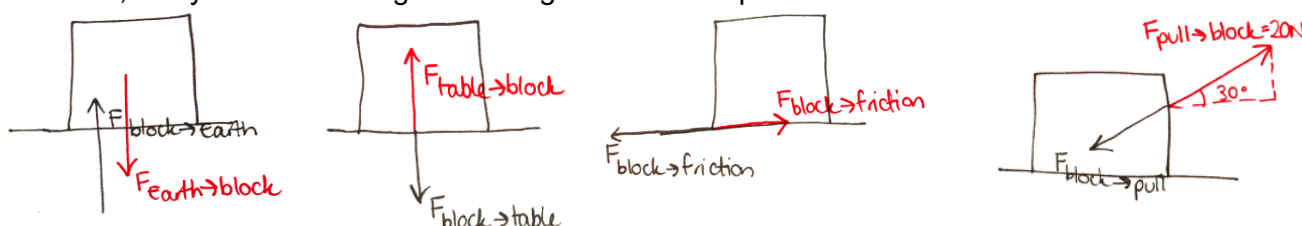


- ⇒ Qualitatively explaining the situation: We know from Newton's First Law that since the block is at rest, there must be a total net force of zero acting on it. So, the force of gravity (exerted by the Earth on the block) must balance the contact force of the table on the block. Similarly, the force of the pull must be balanced by the force of the friction on the block.

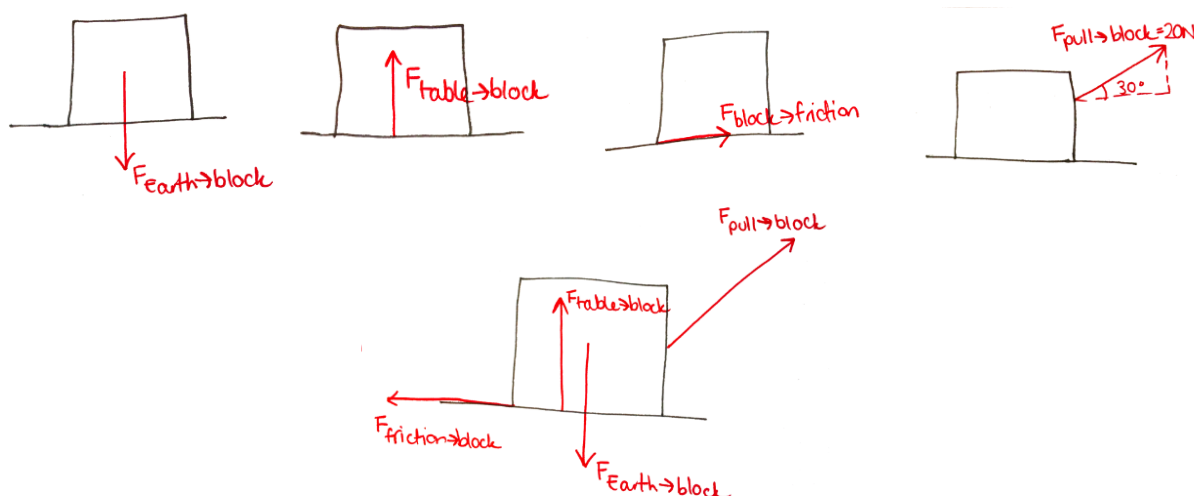
EXAMPLE 2

A block of mass 2kg, is being pulled along a table. The pulling force is 20N at an angle of 30° to the horizontal, as drawn below. Add in all the other forces acting on the block, assuming there is a force due to friction of 10N. What is total horizontal acceleration? Resolve the vertical forces and explain why the block doesn't fly upwards.

- ⇒ So, firstly we draw a diagram adding all the forces present.



We have the force due to gravity, $\vec{F}_{\text{Earth} \rightarrow \text{block}}$, and its force pair the force of gravity exerted by the block on the Earth $\vec{F}_{\text{block} \rightarrow \text{Earth}}$. The contact force exerted by the table on the block, $\vec{F}_{\text{table} \rightarrow \text{block}}$, the contact force of the block on the table, $\vec{F}_{\text{block} \rightarrow \text{table}}$, and we have the force due to friction exerted by the table on the block $\vec{F}_{\text{friction} \rightarrow \text{block}} = 10\text{N}$ which has a force pair $\vec{F}_{\text{block} \rightarrow \text{table}} = -10\text{N}$. And finally, the force of the pull on the block, $\vec{F}_{\text{pull} \rightarrow \text{block}} = 20\text{N}$ (at 30°) and its pair the force of the block on the pull $\vec{F}_{\text{block} \rightarrow \text{pull}}$. So, picking out the forces that are acting on the block we are left with: $\vec{F}_{\text{Earth} \rightarrow \text{block}}$, $\vec{F}_{\text{table} \rightarrow \text{block}}$ (contact force), $\vec{F}_{\text{friction} \rightarrow \text{block}} = 10\text{N}$ and $\vec{F}_{\text{pull} \rightarrow \text{block}} = 20\text{N}$ (at 30°).



⇒ Now, we need to resolve each of the forces into their components. Since the force due to gravity on the block and the contact force of the table on the block cancel each other out, we will ignore them in our calculations

So, we only need to calculate the vertical and horizontal component of the pulling force and the friction force on the block using trigonometry and use $\vec{F} = m\vec{a}$ to calculate the acceleration.

$$\vec{F}_x = -\vec{F}_f + \vec{F}_{px}$$

$$\vec{F}_{px} = 20 \cos(30)$$

$$= 17 \text{ N}$$

$$\vec{F}_f = 10 \text{ N}$$

$$\vec{F}_x = 17 - 10$$

$$= 7 \text{ N}$$

$$\vec{F}_x = m\vec{a}_x$$

$$7 = 2\vec{a}_x$$

$$\vec{a}_x = 3.5 \text{ ms}^{-2}$$

⇒ The horizontal acceleration is $\sim 3.5 \text{ ms}^{-2}$.

⇒ As for the vertical acceleration. We know the block won't fly upwards if we pull it at a slight angle, so our total vertical forces must be balanced:

$$\vec{F}_y = -\vec{F}_g + \vec{F}_{\text{table} \rightarrow \text{block}} + \vec{F}_{\text{pull} \rightarrow \text{block}} \sin(\theta)$$

$$= -\vec{F}_g + \vec{F}_{\text{table} \rightarrow \text{block}} + 20 \sin(30)$$

$$= 10 \text{ N} - \vec{F}_g + \vec{F}_{\text{table} \rightarrow \text{block}}$$

⇒ Since the force due to gravity of the Earth on the block is a constant and won't change but we are adding the vertical force due to the pull on the block, the contact force from the table on the block must decrease to ensure everything is balanced. This means that the block won't accelerate upwards, however, it will sit on the table with less force.

$$\Rightarrow 0 = -\vec{F}_g + \vec{F}_{\text{table} \rightarrow \text{block}} + 10 \text{ N}$$

$$\Rightarrow \vec{F}_g = 10 \text{ N} + \vec{F}_{\text{table} \rightarrow \text{block}}$$

Elastic Collisions

CONTENT

Before we look at elastic collisions, we are first going to take a step back and look at **kinetic energy** and **momentum**. The **kinetic energy** relates to the energy of motion of an object, it can be calculated using:

$$\vec{E}_K = \frac{1}{2}m\vec{v}^2$$

Where \vec{E}_K is the kinetic energy, m is the mass of the object and \vec{v} is the velocity of the object. So, an object that is travelling faster has more kinetic energy than an object of the same mass travelling slower.

Newton described the **momentum** of an object as a quantity of motion. It depends on the mass and motion of the object and can be calculated using the formula:

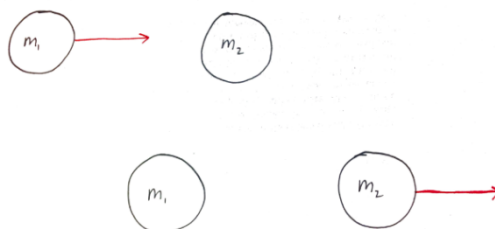
$$\vec{p} = m\vec{v}$$

When objects collide or crash into each other, we use the kinetic energy and momentum of the system before and after the crash to help understand what happened. An **elastic collision** occurs if the total kinetic energy of the system before and after the crash is the same. To calculate the total kinetic energy of the system, we add the kinetic energy of each object:

$$\sum \frac{1}{2}m\vec{v}^2$$

Where the \sum means to sum, so in this case, we add the kinetic energy for each object together.

As a simple example, consider two balls on a pool table. A red one is approaching a yellow one with a constant velocity \vec{v} , the yellow one is stationary. The two balls collide and afterwards, the red one is stationary and the yellow one is moving away with the same velocity of the red ball before the collision. Essentially, the balls have switched velocities, so the kinetic energy of the system before the collision and after the collision is the same, making it an elastic collision.



In all collisions, the total momentum of the system is also conserved. Like with kinetic energy, to calculate the total momentum of the system we add the momentum of each object together.

$$\sum m\vec{v}$$

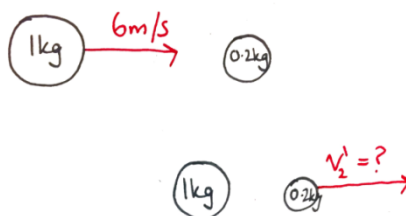
The total momentum for the system before the crash will always be the same as the total momentum after the collision.

EXAMPLE

Two balls are about to have a head on collision. The first particle has a mass of 1kg travelling with a velocity of 6m/s right. The other has a mass of 0.2kg and is at rest. Their collision is perfectly elastic

and the first ball is at rest after the collision while the second is travelling with an unknown velocity. What is the final velocity of the second ball?

Firstly, we begin with a diagram of the system before and after the crash.

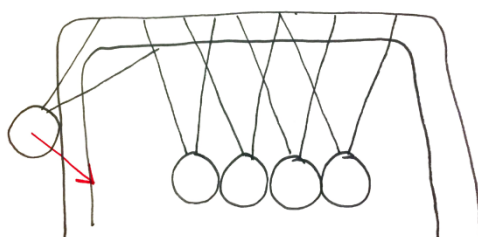


Now, we use the **conservation of momentum** and calculate the momentum before and after the crash:

$$\begin{aligned}
 K_{\text{before}} &= K_{\text{after}} \\
 \sum \frac{1}{2}mv^2 &= \sum \frac{1}{2}mv^2 \\
 \cancel{\frac{1}{2}m_1v_1^2} &= \cancel{\frac{1}{2}m_2v_2^2} \\
 1 \times (6)^2 &= 0.2v_2'^2 \\
 36 &= 0.2v_2'^2 \\
 180 &= v_2'^2 \\
 v_2' &= 13\text{ms}^{-1}
 \end{aligned}$$

EXAMPLE

Newton's cradle is a device with (usually) 5 balls on a held up by a wire just touching each other (see the diagram below). When one ball on the outside is raised and left to fall into the remaining 4 balls, when it crashes into them, it remains motionless while the ball on the other end starts to swing upwards and away from the other balls. This begins a chain reaction where the swinging ball is passed between the two on the end. Using the conservation of momentum and elastic collisions, explain what is happening. What would happen if two balls were initially raised and crashed into the other remaining balls?



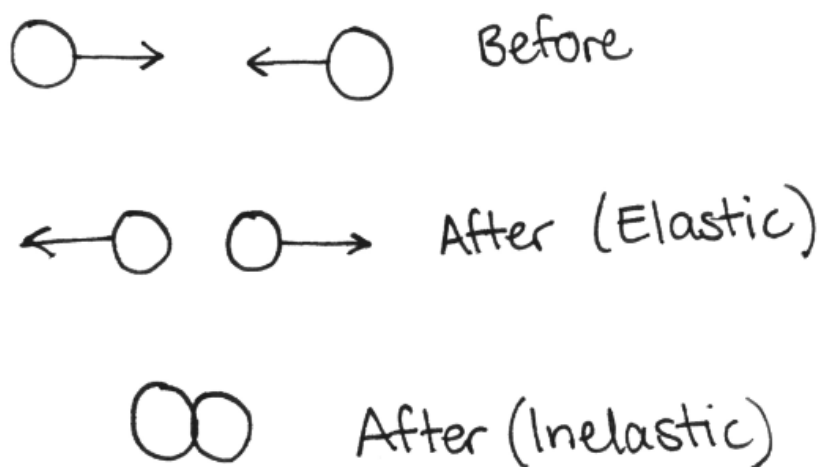
- ⇒ For simplicity, we shall label the balls from left to right as 1-5. So, in the diagram, ball 1 is swinging towards ball two about to crash into it. When ball 1 crashes into ball 2 it remains stationary. Due to conservation of momentum, the momentum of ball 1 must have been transferred into ball 2 (since it cannot be lost) but since ball 2 cannot swing it immediately 'crashes' into ball 3 and transfers the momentum to ball 3. This reaction continues from ball 3 to ball 4 and then when ball 4 'crashes' into ball 5, ball 5 is able to swing and so we see the momentum being transferred from ball 1 to ball 5 and ball 5 begins to swing seemingly magically. If 2 balls were swung at the beginning we would see the same transfer of momentum except both ball 4 and ball 5 would swing instead of only ball 5.



ELASTIC AND INELASTIC COLLISIONS

CONTENT

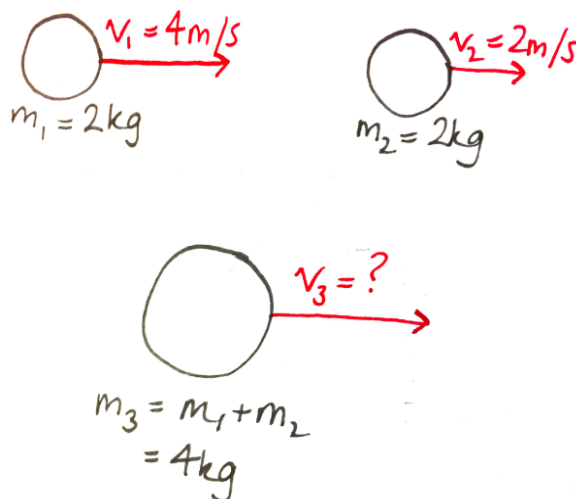
If the total kinetic energy of the system before and after a crash is the same, the crash is called an **elastic collision**. If the kinetic energy isn't the same before and after, the crash is called an **inelastic collision**. The 'missing' kinetic energy has actually been converted to other forms of energy. These include sound, heat and light. As an example of the most extreme inelastic collision, imagine two balls moving towards each other, like in the diagram below. In a perfectly elastic collision, these balls would bounce off each other and move away with the same total kinetic energy. In a perfectly inelastic collision, the two balls stick together.



In both elastic and inelastic collision, the **momentum** is the same before and after the collision.

EXAMPLE 1

A ball with a mass of 2kg is travelling with a velocity of 4m/s to the right. It is travelling towards another ball with the same mass. The second ball is travelling with a velocity of 2m/s to the right. When these two balls collide, they stick together and start travelling together. Using the conservation of momentum, calculate the velocity of the combined two balls after the collision. Is this collision elastic or inelastic?





$$\begin{aligned} p_{\text{before}} &= m_1 \vec{v}_1 + m_2 \vec{v}_2 \\ &= 2 \times 4 + 2 \times 2 \\ &= 8 + 4 \\ &= 12 \text{ kg m/s} \end{aligned}$$

- ⇒ We start with the conservation of momentum. First, we calculate the total momentum before the collision:
- ⇒ Now, we know the mass of the two balls after the collision is just the sum of the mass of the two balls. We also know, by the conservation of momentum, the momentum after the collision must be 12 kg m/s. So, we solve the equation for the velocity of the two balls:

$$\begin{aligned} p_{\text{after}} &= 12 \text{ kg m/s} \\ &= m_3 \vec{v}_3 \\ &= 4 \vec{v}_3 \\ \Rightarrow 12 &= 4 \vec{v}_3 \\ \Rightarrow \vec{v}_3 &= 3 \text{ m/s} \end{aligned}$$

- ⇒ Now, to determine if the collision was elastic or inelastic. To do this, we calculate the kinetic energy before and after the collision and see if they are equal.
- ⇒ Since the kinetic energy before and after isn't the same, the collision is inelastic.

$$\begin{aligned} \vec{E}_{k-\text{before}} &= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \\ &= \frac{1}{2} \times 2 \times (4)^2 + \frac{1}{2} \times 2 \times (2)^2 \\ &= 16 + 4 \\ &= 20 \text{ J} \end{aligned}$$

$$\begin{aligned} \vec{E}_{k-\text{after}} &= \frac{1}{2} m_3 v_3^2 \\ &= \frac{1}{2} \times 4 \times (3)^2 \\ &= 18 \text{ J} \end{aligned}$$

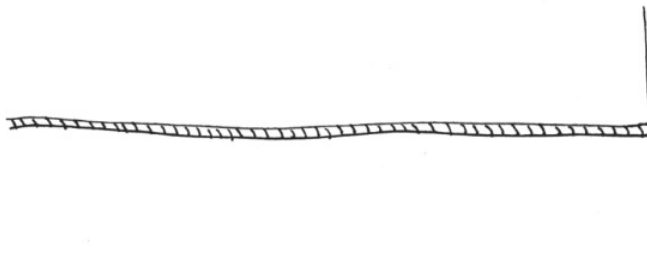
$$\vec{E}_{k-\text{before}} \neq \vec{E}_{k-\text{after}}$$



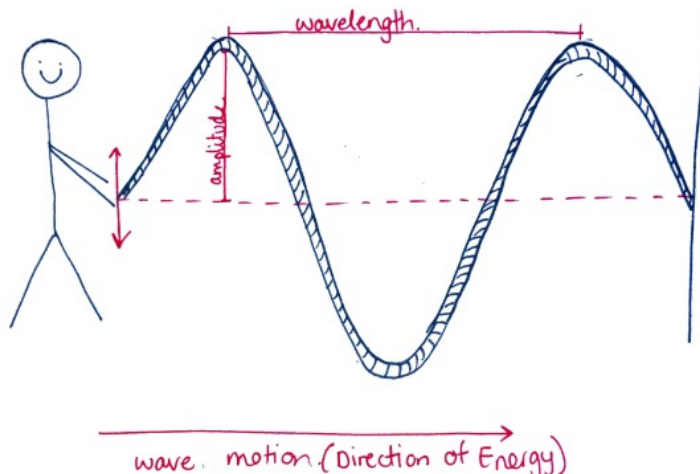
WAVE PROPERTIES

CONTENT

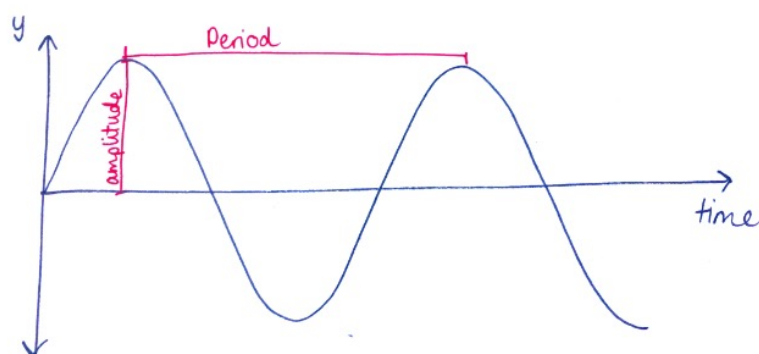
Waves transfer energy and can come in many different shapes. Some waves need a medium (e.g. a material) for propagation, whilst some waves can travel in a vacuum (i.e. do not require a medium). We can understand the type of wave and the way it transfers energy by plotting the wave: a plot of the displacement as a function of time (called the displacement-time graph) and a plot of displacement and the position of the wave (called the displacement-position graph). Consider a rope where one end is fixed to a wall. When the rope isn't being moved up or down it is just a flat line, this is its equilibrium position, as shown below.



When side not attached to the wall is moved up and down, a wave is created, as seen below.



To plot a displacement-time graph, we consider one point on the wave. Then we plot how that point moves from the equilibrium position over time. So, for our rope example, the displacement as a function of time graph involves plotting time on the x-axis and displacement on the y-axis, as shown below:



Displacement-time graph

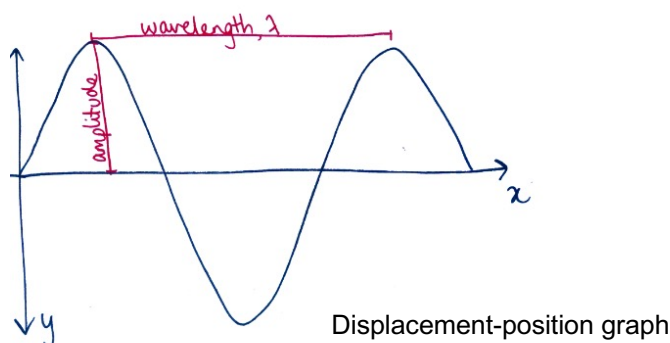
When plotting the position of the wave, we plot the y-position of the wave against the x-position of the wave (see graph below). For the rope example, the graphs look very similar but they are actually showing different information.

There are 6 main properties of waves that we can calculate from these graphs:

- **Velocity (v)**
The velocity of the wave, calculated by multiplying the frequency and the wavelength together (m/s)
- **Frequency (f)**
The frequency is the number of waves that pass through a fixed point every second (Hz)
- **Period (T)**
The period is the time taken for one full oscillation to complete (s)
- **Wavelength (λ)**
Wavelength is the distance of one full oscillation of the wave (m), easily measured from crest to crest or trough to trough
- **Amplitude**
The amplitude is the height of the wave, the maximum displacement from the equilibrium position (m)

On the displacement-time graph, we can determine the amplitude and the period. To calculate the amplitude, we measure the distance from the $y=0$ to the first crest, as shown in the graph. The period can be measured from finding the difference between two closest identical points on the wave (e.g. from crest to crest or trough to trough).

On a displacement-position graph, we can determine the amplitude of the wave as well as the wavelength. The wavelength is found using the same technique as the period but since we are now plotting the x-position on the x-axis, not the time, the wavelength can be easily calculated by measuring the distance between two adjacent crests or troughs.



Note that we cannot measure the frequency or period directly from a displacement-position graph. To calculate the frequency of the wave, we can use two equations - either the equation for velocity ($v = f\lambda$) or that of the period. The frequency and period are inversely related. So, in addition to the aforementioned equation, we can also calculate the frequency using the equation:

$$f = \frac{1}{T}$$



EXAMPLE 1

A wave is travelling with a frequency of 50Hz and a wavelength of 0.25m. What is the velocity of the wave?

- ⇒ Let us start by writing all the known information, and what we wish to calculate to determine the appropriate formula:

Variable	Value
Frequency (f)	50 Hz
Wavelength (λ)	0.25m
Velocity (v)	???

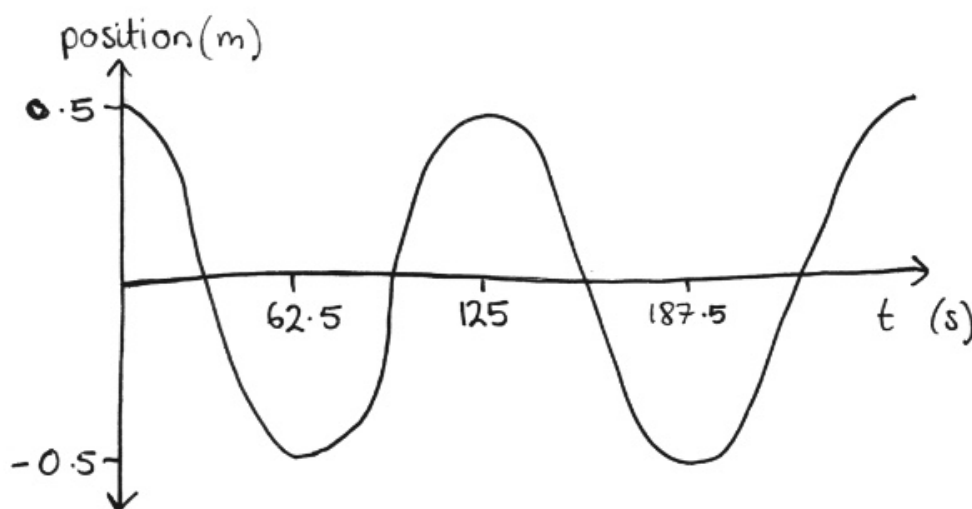
- ⇒ Now it is clear that we need to use $v = f\lambda$ as the equation for velocity.

$$\begin{aligned}v &= f\lambda \\&= 50 \times 0.25 \\&= 12.5 \text{ m/s}\end{aligned}$$

- ⇒ So, the velocity is 12.5m/s in the positive direction.

EXAMPLE 2

Using the displacement-time graph below, determine the frequency of the wave.



- ⇒ Firstly, we decide which formula to use. Since we want to calculate the frequency and have a displacement-time graph, we will use the formula $f = \frac{1}{T}$.
- ⇒ Now, from the graph we can see the period is 125s (difference between two adjacent crests), thus we can calculate the frequency as follows:

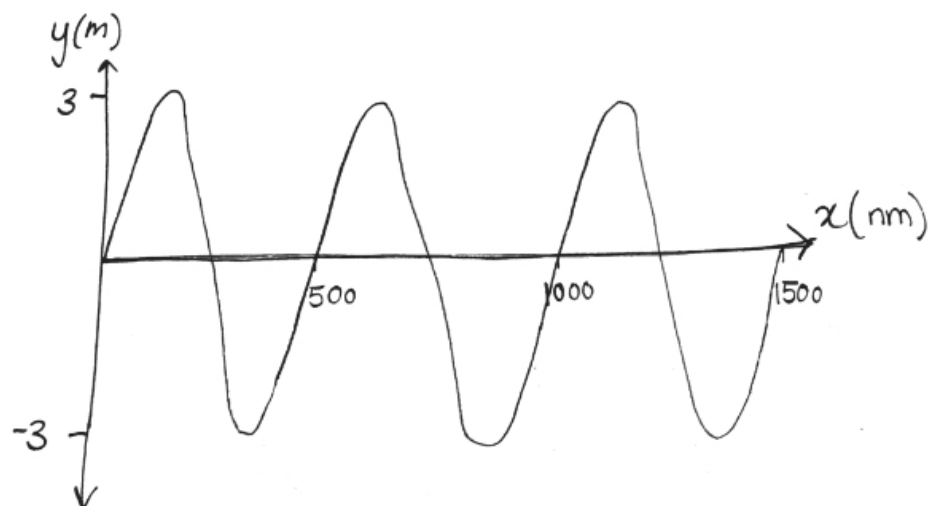
$$\begin{aligned}f &= \frac{1}{T} \\&= \frac{1}{125} \\&= 0.008 \text{ s}^{-1} \\&= 0.008 \text{ Hz}\end{aligned}$$



EXAMPLE 3

A periodic wave has a wavelength 500m and amplitude 3m. Plot a displacement-position graph for the wave.

⇒ Since we are plotting a displacement-position graph, we know our axis are going to be the x-position and y-position:





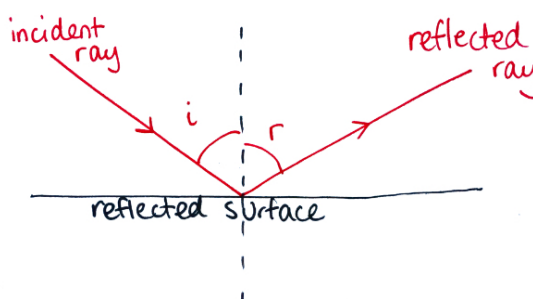
WAVE BEHAVIOURS

CONTENT

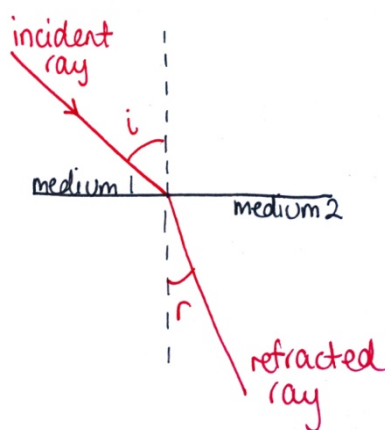
Waves behave differently when they reach a barrier or a new medium. These behaviours can be sorted into **reflection**, **refraction**, **diffraction** or **superposition**. We will be looking at superposition later.

When a wave encounters a barrier, the wave can be **reflected**. This means the wave 'bounces' off the barrier and changes direction. An example of this is when light hits a mirror. If you shine a torch at a mirror the light reflects off the surface and you can see a 'picture' of the torch in the mirror.

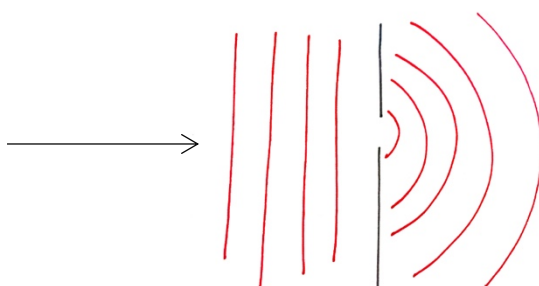
When a ray hits the surface, this is called the incident ray and the angle the ray hits the surface is called the angle of incidence, i . The angle of incidence is measured from the normal to the surface. Once the ray is reflected it is called the reflected ray and the angle of the ray is called the angle of reflection, r . This is also measured from the normal of the surface. The angle of incidence is equal to the angle of reflection.



Sometimes, instead of being entirely reflected, a wave can pass through into a different medium and it is **refracted**. Refraction is when the light 'bends' in the new medium. Similar to a reflected wave, there is also an incident ray and angle of incidence as well as a refracted ray and angle of refraction. The only difference is the refracted ray is not entirely reflected off the surface, instead travels through the new medium.

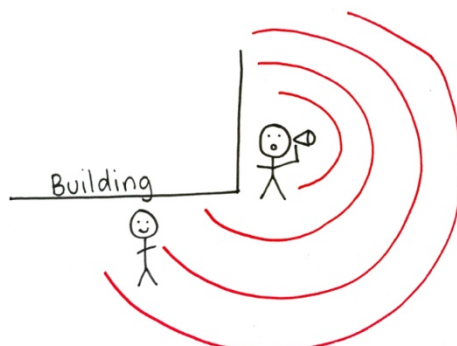


When waves pass through an opening they can also bend around the obstacles or change directions. This is called **diffraction**. For example, if straight water waves are travelling through the water and then pass through a slit the waves change direction and the water waves spread out, like in the diagram below.





Waves can also be diffracted around an obstacle. For example, if you are standing next to a building and shouting into a megaphone, around the corner, your voice can be heard. This is because the sound waves are being bent around the corner of the building.



The angle of diffraction is directly proportional to wavelength. The longer the wavelength, the greater the amount of diffraction, i.e. the more the wave can 'bend' around the obstacle.

EXAMPLE 1

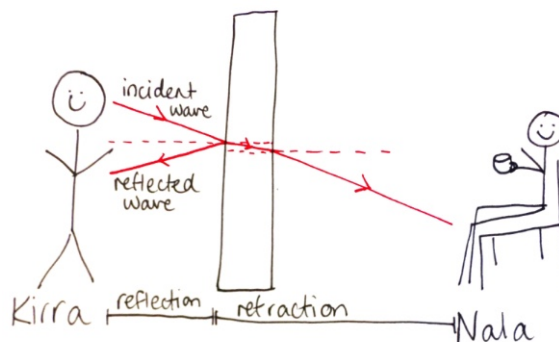
If a hiker is lost in a forest and wishes to send out a distress sound, should they use a high frequency or low frequency sound (assuming the waves have the same velocity) to bend around the trees in the forest? Explain your reasoning.

- ⇒ Given the hiker needs the signal to travel as far as possible and avoid being blocked by a large tree, we need to determine which of the two sound signals can 'bend' around the trees more. So, let us look at how the wavelength is related to the frequency so we can determine which signal will be able to diffract around larger trees.
- ⇒ We know the equation for velocity of a wave is $v = f\lambda$ and we also know the velocity of the waves are the same. If we increase the frequency of the wave, the wavelength must decrease to keep the velocity constant. Likewise, if we decrease the frequency of the wave, the wavelength must increase.
- ⇒ This means, the lower the frequency, the longer the wavelength.
- ⇒ So, the hiker is best sending out a low frequency (longer wavelength) signal because the longer the wavelength, the more the sound waves can 'bend' around the trees.

EXAMPLE 2

Kirra is looking at a window of a café, trying to see if her friend, Nala, is already waiting inside. Kirra notices she can see Nala inside but can also see her reflection in the window. Explain this behaviour of light.

- ⇒ Since Kirra can see both Nala and herself in the window, there must be a combination of behaviours involved.
- ⇒ Firstly, Kirra can see herself which means that at least some of the incident light wave is being reflected. However, since Kirra can also see Nala inside the café, light waves must also be travelling through the

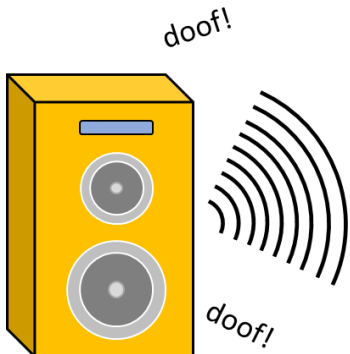
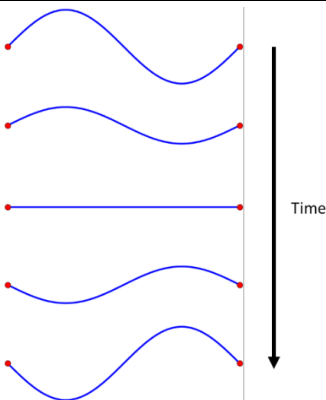


glass (being refracted). This is demonstrated in the diagram below. This is called partial reflection and refraction.

WAVE SUPERPOSITION

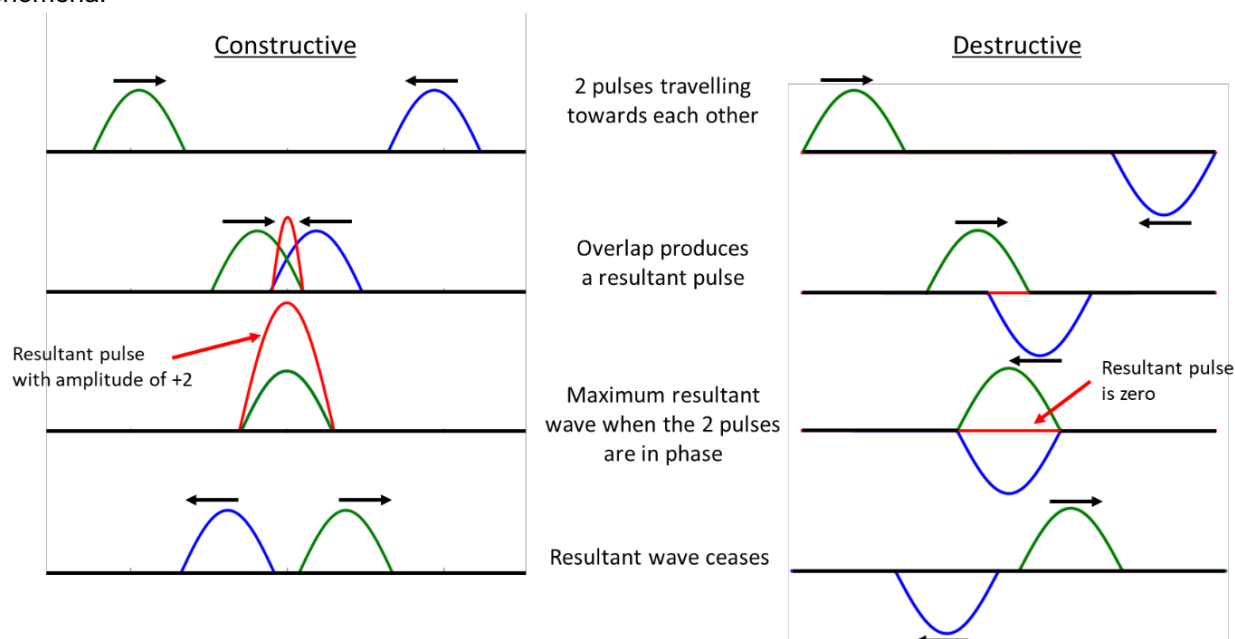
CONTENT – TRAVELLING AND STANDING WAVE

When we think of waves, we associate the word with waves on the beach or a Mexican wave. The definition of waves in physics states that waves are oscillations that propagate in space and matter. The most common type of waves we encounter in our daily lives are mechanical waves. These include sound waves and wave motion in a string. Sound waves are an example of a travelling wave whereas a wave propagating in a string is an example of a standing wave. The position of a standing wave does not change while a travelling wave moves through space and matter.

Travelling wave	Standing wave
	

CONTENT – WAVE SUPERPOSITION

When two waves overlap, they combine to produce a resultant wave. This phenomenon is called the *principle of superposition* or *wave superposition*. The amplitude of the resultant wave increases when the two interacting waves are in phase, and it decreases when they are out of phase. To illustrate, consider two pulses with an amplitude of +1 travelling towards each other. When the two pulses start overlapping, the amplitude of the two pulses starts adding up following the superposition principle. When the two pulses reach maximum overlap (i.e. in phase), the resultant pulse will have an amplitude of +2. This behaviour is known as *constructive interference*. When the pulses have amplitudes in opposite directions, as shown in the diagram, the resultant pulse will have an amplitude of zero. This behaviour is known as *destructive interference*. The diagram illustrates the two phenomena.



REAL WORLD EXAMPLE

A real-world example of wave superposition at play is in musical instruments like the guitar. If you pluck the first string (string *E*: the thinnest) on the guitar, the string will vibrate and produce a sound. If you then pluck the second string (string *B*), you will create another sound different to the first string. Suppose now you pluck both string *E* and *B* at the same time, the sound that you will hear will be different than that of the sound produced by plucking strings *E* and *B* separately. This is because the sound wave generated by strings *E* and *B* add up to create a resultant sound wave (wave superposition). Different sounds can be created through a combination of strings and notes when you put your fingers on the guitar neck. These sets of notes are known as guitar chords.

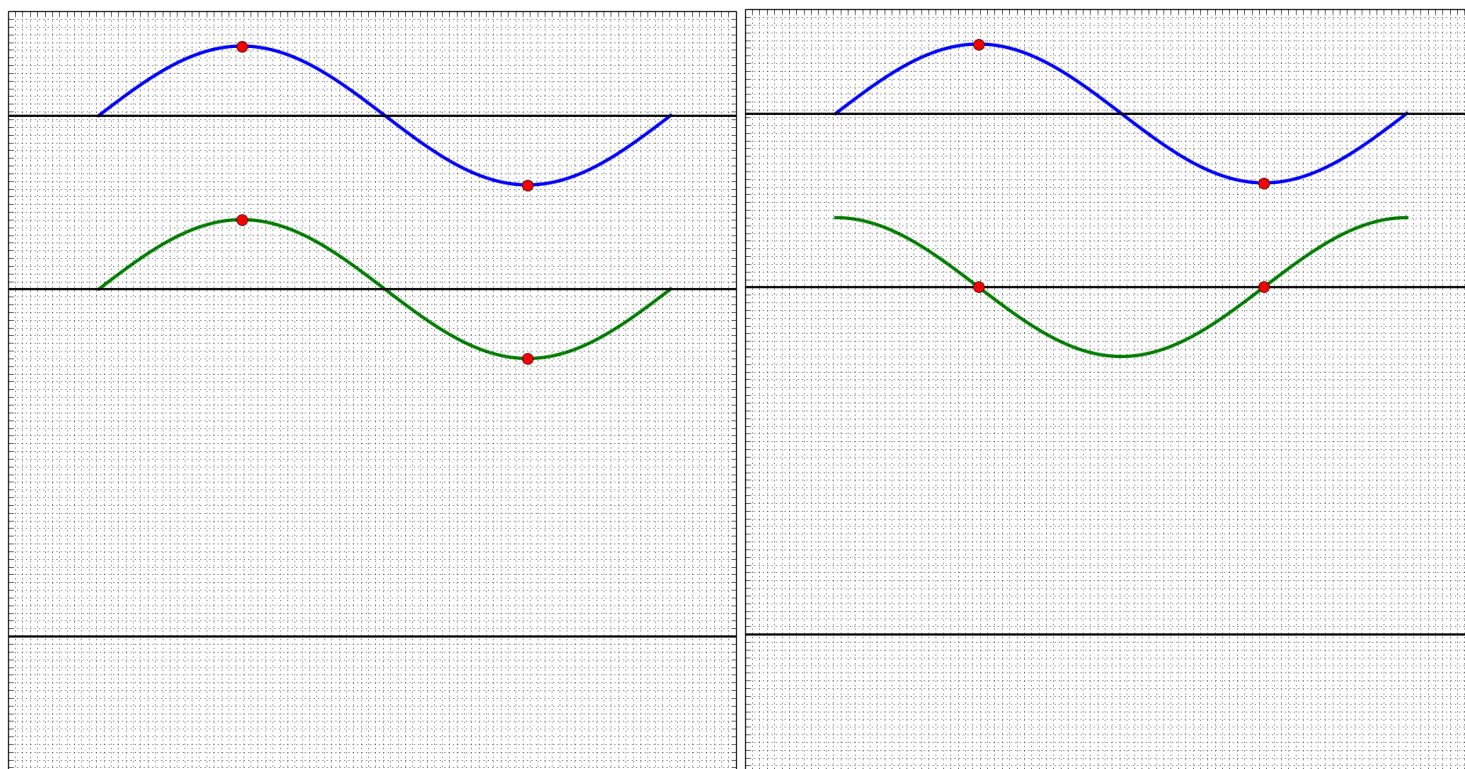


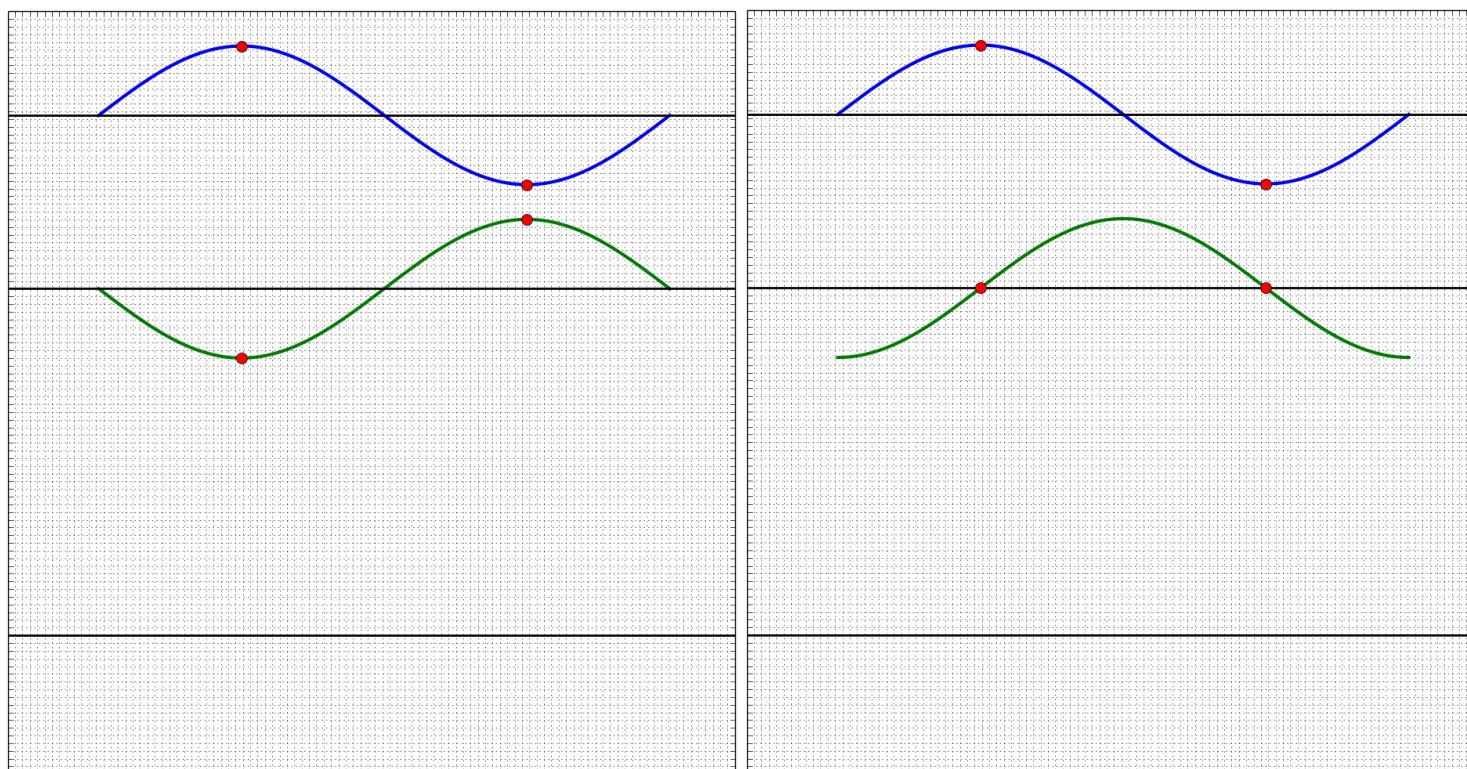
QUESTION – INTERNET RESEARCH

How do noise-cancelling headphones work?

QUESTION – PENCIL

Using the principle of superposition draw the resultant wave produced by the two standing waves below.





Investigation: Wave Behaviours

INTRODUCTION

It's hard to find a playground that doesn't have a swing set and swinging is always heaps of fun. But what makes the swing go higher and faster? How can we push someone so that they swing higher and have a better time?



We know that when we push someone on a swing it is best to push them from when they are highest and on their way back down to the ground. But how often should you push them so that the person swinging goes as high as possible?

In this experiment, we will try to analyse the motions of a swing using a pendulum. This is a mechanical system and the pendulum is a harmonic oscillator. We will be measuring the natural frequency of a pendulum and determining the frequency of pushes that improve the swinging motion of the pendulum the most. We will also be considering the energy transformations of this system. Things to think about before the experiment: at what point of a swing do you think you are moving the fastest?

1. QUESTIONING AND PREDICTING

So, let us think about the aim of this investigation.

1. What is the natural frequency the pendulum wishes to swing at?
2. What frequency of an added regular force increases the swinging motion the most?

HYPOTHESIS

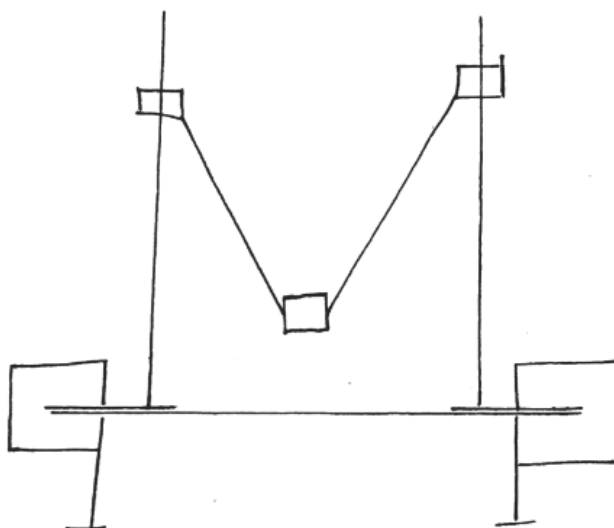
The regular pushing force will be most effective at increasing the swinging motion at a frequency (that matches the natural swing/is fastest/is slowest)

2. PLANNING INVESTIGATION

This investigation has been planned for you.

We will use one pendulum and find the frequency of pushes that naturally increases the amplitude of the swinging pendulum.

1. Set up your apparatus as shown in the picture below:



2. Place a 1 meter ruler horizontally against the wall. This will give you a scale for measuring the displacement of the object.
3. Use a smartphone or digital camera to record a video of the mass swinging from the time when it is released.
4. Raise one mass up a small angle and release it to let it swing.
5. The frame rate (in frames per second) of the recording is a fixed value which you can get from the camera settings. Find this value. Convert this to the time per frame. (For example, if the video is recorded at 30 fps, each frame lasts for $1/30^{\text{th}}$ of a second.)
6. Look at the video frame by frame. What is the position of your object against each ruler for each frame?
7. Record the object's horizontal position and vertical position at each frame in the table on the next page. If there are a large number of frames, you may wish to take data every five or ten frames.
8. Repeat the experiment but 'bump' the mass at a consistent rate and force so that the pendulum keeps swinging smoothly and the amplitude of the pendulum increased. Make sure the hand is only moving in the horizontal direction.

Some groups may choose to download the free app 'VidAnalysis Free' for androids or the app 'Vernier Video Physics' for iPhones and analyse the video within the app to create graphs of the x-positions and y-positions over time.

3. CONDUCTING INVESTIGATION

For each pendulum, you should take a video that clearly shows the mass swinging and the scale of the ruler until the object hits the floor and the motion of the hand hitting the mass. Choose suitable time intervals and record the horizontal displacement of the mass and the horizontal displacement of the hand when applicable.

No input:

Time (s)	Horizontal Displacement (m)	Any other comments on motion of object and taking the video

Input frequency:

Mass displacement:

Time (s)	Horizontal Displacement of mass (m)	Any other comments on motion of object and taking the video

Hand displacement:

Time (s)	Horizontal Displacement of hand (m)	Any other comments on motion of object and taking the video

Did you make any changes to the method? Did you have design problems to solve? Did you have some 'smart' ways of doing the investigation?

4. PROCESSING AND ANALYSING

Plot the displacement of the mass versus time on graph paper or using a spreadsheet program for the horizontal displacement. Plot the displacement of the hand (when used) versus time as well.

What shape are each of the displacement-time graphs of the mass and the hand? What shape would you expect the velocity-time graphs to be (remember the velocity is the change in displacement over the change in time)?

Try plotting the velocity-time graph of the pendulum mass for two periods of oscillation.

Finding the frequency of oscillation requires us to know the period of the oscillation. Mark on your graph a point where the displacement is a maximum and record the value of the time at that point, then record the time at the next maximum point on the graph. What is the difference between those times? To improve the validity of this period, record the time it takes for your pendulum to complete 10 full oscillations. Using the formula,

$$T = \frac{t}{10},$$

where T is the period of one oscillation and t is time for 10 full oscillations, calculate the time for one oscillation.

Repeat this for the input force of the hand too.

What shape do you expect the velocity-time plots to be for both the mass and the hand?

5. PROBLEM SOLVING

- ⇒ Consider the energy transformations (always has constant mechanical energy, but the kinetic is max when position is zero and zero when still at max distance vice versa for the potential energy)

This is the section where we think about the results and what they mean.

When we let the pendulum swing it has a periodic motion. Each pendulum has a natural frequency at which it wants to oscillate. We can calculate it using:

$$f = \frac{1}{T}$$

where f is the frequency and T is the period. The period is measured in seconds as the time for the pendulum to complete one oscillation (the time we measured in our analysis)

What was the natural frequency of the pendulum?

What was the frequency of the optimal input force?

What can you say about these two frequencies?

The **amplitude** of the pendulum is the maximum displacement of the mass during the oscillation.

The **natural frequency** of oscillation is the frequency your pendulum wishes to oscillate at. No matter how high you begin the oscillation at or how hard you push the mass at the beginning, it will always oscillate with this frequency.

The **driving frequency** of a harmonic oscillator is the frequency of the input force, how frequently you pushed the pendulum in this case.

What relationship did you observe between the driving frequency and the natural frequency which increased the amplitude of your pendulum?

6. CONCLUSIONS

The natural frequency of the pendulum was _____.

The frequency of the driving force that increased the amplitude was _____.

The total energy of the system stays the same and the kinetic and potential energies _____.

Investigation: Interference of Sound Waves

INTRODUCTION

A speaker produces sound waves using a vibrating cone which changes the pressure of the air around it. The wave travels outwards in every direction and eventually reaches your ears.

Sound waves have the same properties as other types of waves, such as reflection and interference. Interference occurs when two waves are passing through the same position. These can be waves from two different sources, or a wave from one source plus its reflection. Both constructive and destructive interference are possible. This will affect the loudness of the sound you hear.

In this experiment, we will investigate the interference of sound waves produced by speakers and how the result varies with position. We will use the interference pattern to find the wavelength of a tone and compare it with the theoretical wavelength.

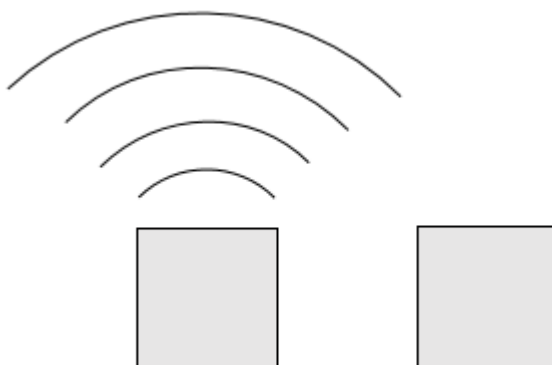
1. QUESTIONING AND PREDICTING

Let us think about the aim of this investigation.

1. What conditions are needed to produce interference between sound waves?
2. Where will constructive interference occur?
3. Where will destructive interference occur?

This diagram shows the wavefronts coming out of a speaker. Sketch what happens as they get further from the speaker.

Sketch the wavefronts coming from the second speaker (which is playing the same sound) and describe the results of the interference.



HYPOTHESIS

Regions where the waves interfere constructively will sound _____ (louder/quieter).

Regions where the waves interfere destructively will sound _____ (louder/quieter).

The spacing of maxima and minima depends on _____.

2. PLANNING INVESTIGATION

This investigation has been planned for you. It is most suited to being performed by a whole class.

Part A: Interference between two sources

You will need a pure (sine wave) tone to be played through two speakers. This can be created by a signal generator, or by an online app/recording. The frequency should be about 340 Hz in order to produce a wavelength of about 1 metre – this will make it easier to find distinct points of interference.

1. Place the speakers on a desk, facing in the same direction about a metre apart, and play the tone.
2. Have one student stand approximately the same distance from both speakers, at least a metre away. The student should find the location where the resulting tone sounds the loudest and place masking tape or a chalk mark on the floor¹.
3. Get a second student to start from the first mark² and find a nearby point where the sound is loudest. Mark the location.
4. After 5-10 points of constructive interference have been found, repeat steps 2-4, this time finding locations where the sound is quietest. Mark these locations so they can be distinguished from the maxima.

Part B: Interference with a reflection

For this section you will need only one speaker, plus a strongly reflective surface such as a sheet of metal or tile. You should also use a small power meter, such as a smartphone app, to measure the resulting volume in order not to block the wave.

1. Place the tile approximately two wavelengths away from the speaker and play the tone.

¹ If it is difficult or impractical for students to find regions of constructive/destructive interference by ear, a power meter can be used to measure the loudness in decibels at different locations. Smartphone apps can perform this task.

² Tip: Ensure all students listen to/measure the volume at approximately the same height above the ground; sound waves travel in three dimensions and we are interested in the 2D pattern at one height.

2. Move the sensor/meter along the path of the sound to find the loudest and quietest regions.
3. Adjust the position of the reflector until the greatest difference between maximum and minimum volume is observed. This indicates that a standing wave has been formed.
4. Mark the positions of maxima and minima on the floor using masking tape.

3. CONDUCTING INVESTIGATION

After each part, take a photograph or draw a scale diagram showing where the regions of constructive and destructive interference occur as marked by the students. Compare the pattern obtained from Part A of the experiment with the pattern you drew in Section 1. On your photograph/diagram, sketch the locations of wavefronts from each speaker which would give you the pattern.

If time permits, you should perform this investigation for multiple wavelengths in order to compare results.

Did you make any changes to the method? Did you have design problems to solve? Did you have some 'smart' ways of doing the investigation?

4. PROCESSING AND ANALYSING

What do you observe about the spacing of the loudest and quietest points compared to the wavelength of the sound? Do the How do they change with wavelength?

Measure the distance between the wavefronts. How does it compare to the approximate known wavelength of the sound?

Observe the pattern of maxima and minima obtained in Part B of the experiment. What can you say about the spacing between them? The distance between two neighbouring maxima or two neighbouring minima of the standing wave is equal to half the wavelength of the sound wave.

5. PROBLEM SOLVING

One way to think about the wavelength of a sound wave is as the distance between wavefronts. The shorter the wavelength (i.e. the higher the frequency), the further apart the wavefronts are. This will in turn affect the spacing of the interference maxima (completely constructive) and minima (completely destructive).

In Part A of the experiment, we used the characteristics of interference between two waves of the same wavelength in order to locate the wavefronts and hence make an experimental measurement of the wavelength.

In Part B of the experiment, we used the property of reflection to effectively 'create' a second wave travelling in the opposite direction. If we fit a certain number of wavelengths between the source and the reflector, we can create a standing wave where the maxima and minima are always in the same place. An animation of this can be seen at:

https://commons.wikimedia.org/wiki/File:Standing_wave_2.gif

The distance between neighbouring maxima/minima gives half the wavelength.

Because we know the speed of sound in air, knowing the frequency of our generated sine wave allows us to make a theoretical calculation of wavelength using the wave equation:

$$v = f\lambda$$

Where v is the speed of sound (≈ 340 m/s) and f is the frequency in Hz.

The theoretical wavelength can now be compared to the experimentally measured wavelength.

6. CONCLUSIONS

Interference of sound waves with the same frequency can be measured via the loudness of the sound.

The spacing of maxima and minima depends on wavelength. The longer the wavelength, the _____ (closer/further) the maxima and minima.



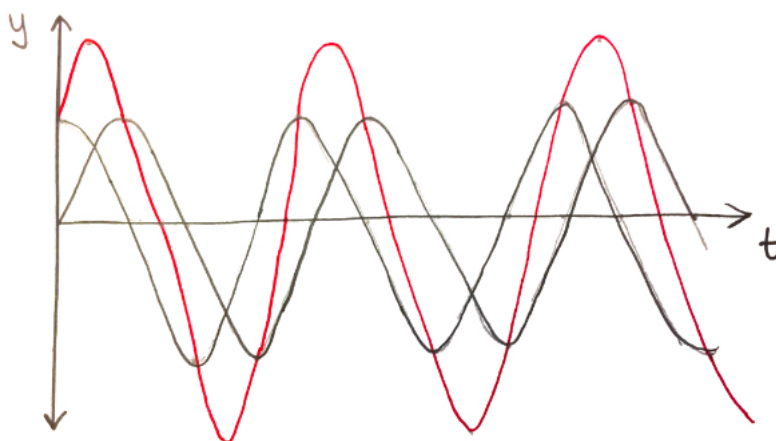
BEATS AND DOPPLER EFFECT

CONTENT

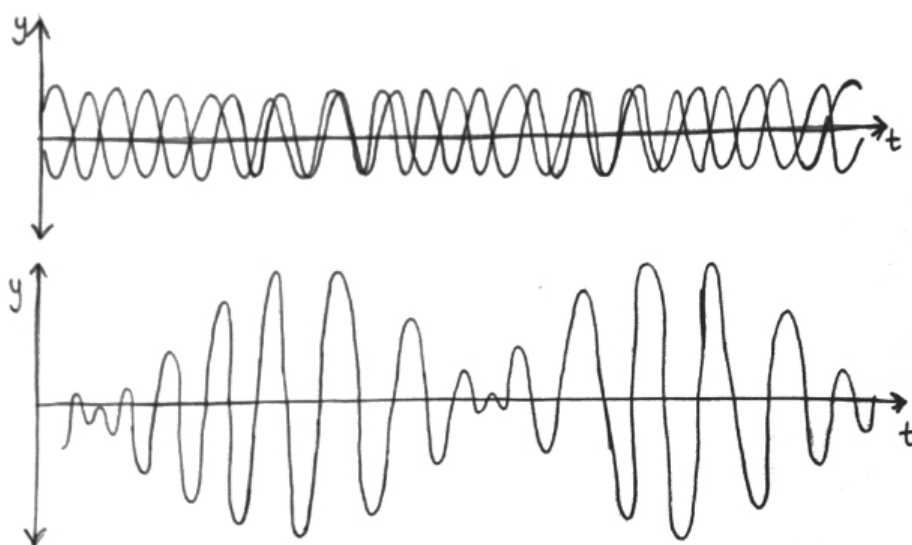
Both the Doppler Effect and Beats are special effects of waves. They occur with all types of waves such as light waves, sound waves and water waves. In this worksheet we will just be working with sound waves, represented as transverse waves on a graph. Here, the crests of the graph are the compressions and the troughs the rarefactions of the sound waves.

Beats are not to do with the beats of a song or the beats of a drum. In fact, beats occur when there are 2 sources of waves with slightly different frequencies. In order to figure out what beats are we have to first revise what the Principle of Superposition is.

When an observer is receiving multiple waves, it can be difficult to identify the waves separately. Instead a resultant wave that is the addition of the two waves is observed, this is calculated by adding the amplitude of each wave. We can see this below, where the red wave is the resultant wave of the two black waves.



In the above example, both the black waves have the same frequency but the crests just arrive at different times. However, in the example below, the two top waves have different frequencies. The two waves then go from destructive interference to constructive and back again. This means the resultant wave goes from a maximum amplitude to no amplitude and back again. In the case of a sound wave, this means the resultant wave sounds like it is getting louder then silent, then loud again, shown in the bottom wave. These are called beats.





The frequency of the beats is related to the frequencies of the two initial waves. We can calculate the frequency of the beats using

$$f_{beats} = |f_2 - f_1|$$

Where f_1 and f_2 are the frequencies of the two initial waves.

EXAMPLE

Nero is trying to tune their guitar. They are playing the open B string (which should be tuned to 247Hz), and the higher G string while holding the 4th fret, this plays the same B note. Nero can hear beats but they know the G string is in tune. The beat frequency is 5Hz. When Nero decreases the frequency of the B string by 5Hz, the beat frequency increases to 10Hz.

What are the two possibilities for the initial frequency of the B string? Should Nero continue to decrease the frequency of the B string in order to tune it?

- ⇒ Firstly, we use the beat frequency formula from above, we know the beat frequency, f_{beat} , is 5Hz and the frequency of the second string (the 4th fret of the G string), f_2 , is 247Hz. So:

$$\begin{aligned} f_{beat} &= |f_2 - f_1| \\ 5 &= |247 - f_1| \\ \pm 5 &= 247 - f_1 \\ \Rightarrow f_1 &= 247 + 5, 247 - 5 \\ &= 242, 252 \end{aligned}$$

- ⇒ Now, since the beat frequency increases when Nero decreased the frequency of the B string, we can determine what the original frequency of the B string was and whether they are tuning in the correct direction.
- ⇒ To do this, we will assume the frequency of the B string is each value calculated in the previous part and subtract 5Hz to calculate the new possibilities for the frequency of the B string. So, after Nero has done the initial changes to the B string, the new frequency becomes either 237 or 247. Now, to calculate the beat frequency assuming each of these frequencies of the B string:

$$\begin{aligned} f_{beat} &= |f_2 - f_1| \\ &= |247 - 237| \\ &= 10 \text{ Hz} \end{aligned} \qquad \begin{aligned} f_{beat} &= |f_2 - f_1| \\ &= |247 - 247| \\ &= 0 \end{aligned}$$

- ⇒ Since the new beat frequency is 10Hz not 0Hz, the initial frequency of the B string must be 242Hz which means Nero is tuning in the wrong direction.



CONTENT

The Doppler Effect is the result of waves being emitted from a moving source, or an observer moving while detecting waves. Consider the example below where the dot in the middle is moving with a velocity of 5m/s to the right and emitting a pulse every 1 second. The velocity of the wave is 10m/s. When $t = 0s$ the pulse emitted 1 second earlier has travelled radially outward 10m. When $t = 1s$, the pulse emitted 1 second earlier has travelled 10m radially outward from where the dot was 1 second ago (the x) but another pulse is emitted from the new location of the dot. Every second, another pulse is emitted from the dot at its new location and the previous pulses continue to travel radially outwards from where they were emitted.

As a result, by the time the pulses reach the observer at the right, the pulses seem to come more frequently. If the observer was standing to the left of the dot and the dot was moving away, it would appear like the pulses are coming less frequently.

This is the Doppler Effect: the *apparent* frequency observed due to a moving source of waves. If the source is moving towards the observer, the frequency increases and if the source is moving away from the observer the frequency decreases. This is easily observed with a train passing a platform without stopping. As the train approaches, the pitch of the train engine is higher (because of a higher frequency) and after the train passes, the pitch decreases (because of the lower frequency). As a result the train makes a neeeeeeee-ooowwww sound as it passes. Astronomers also use the Doppler Effect, they can observe light emitted by distant galaxies become redder (lower frequency) because the galaxy is moving away from the Earth.

We can calculate the apparent observed frequency of a wave using

$$f' = f \frac{v_{\text{wave}} + v_{\text{observer}}}{v_{\text{wave}} - v_{\text{source}}}$$

where v_{wave} is the velocity of the wave, v_{observer} is the velocity of the observer, v_{source} is the velocity of the source emitting the waves, f' is the observed frequency and f is the original frequency emitted.

EXAMPLE

Koramo is an emergency doctor and hears the siren of an approaching ambulance. She knows the ambulance siren has a frequency of 950Hz. The ambulance is travelling towards the hospital at a velocity of 60km/h and the speed of sound in air is 330m/s. Firstly, would you expect Koramo hear the siren with a higher or lower frequency? Secondly, what frequency does Koramo hear as the ambulance approaches?

- ⇒ Firstly, because the ambulance is moving towards Koramo, we expect the frequency to increase
- ⇒ Secondly, we will right all the variables we have so we can sub them into the equation.

Variable	Values
f'	?
f	950Hz
v_{wave}	330m/s
v_{observer}	0m/s
v_{source}	16.7m/s

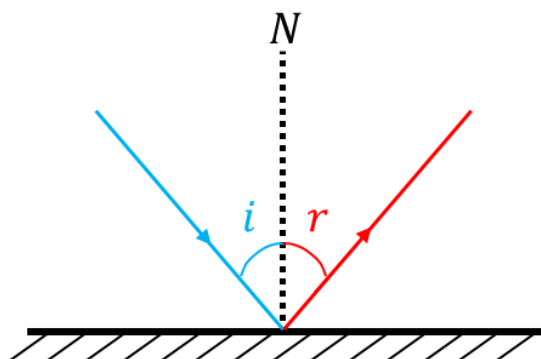
- ⇒ Subbing these values into the equation we find:

$$\begin{aligned}
 f' &= f \frac{v_{\text{wave}} + v_{\text{observer}}}{v_{\text{wave}} - v_{\text{source}}} \\
 &= 950 \frac{330 + 0}{330 - 16.7} \\
 &= 1000.638 \dots \\
 &= 1000 \text{ Hz}
 \end{aligned}$$

RAY MODEL OF LIGHT

CONTENT – REFLECTION

When light strikes a boundary between two materials, some of it will be reflected, and some will be refracted. In the ray model of light, the path light travels in is represented as a straight line which we call a ray. The ray continues in a straight path until it hits the boundary of an interface. When light hits a boundary, at least some of the light is reflected. The Law of Reflection states that the incident ray (incoming ray, marked blue in the diagram on the right), reflected ray (marked red) and the normal, N , lie on the same plane. If the angle of incidence is i with respect to the normal, then the angle of reflection is r and these two angles are equal. Reflection according to the ray model of light is illustrated in the diagram on the right. Reflection can take place from smooth/flat surfaces, or from irregular/bumpy surfaces. In both cases, the rays obey the Law of Reflection. The Law of Reflection is exploited in devices such as mirrors, microscopes and telescope where incoming light bounces off a reflective surface (in the latter two cases inside a cylinder tube).



CONTENT – REFRACTION

Refraction is the bending of a light ray as it passes from one medium to another. The ray bends towards the normal if its speed decreases as it moves into a different medium, whereas it bends away from the normal if its speed increases with a change in medium. As explained in detail below the refractive index is essential for determining the bending of light towards or away from the normal.

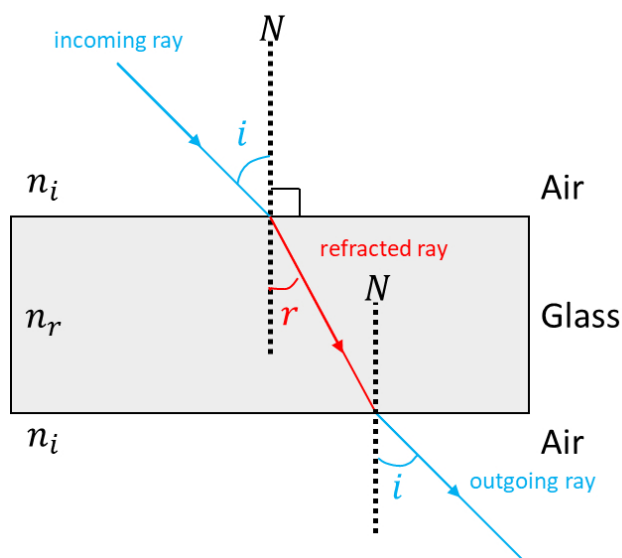
The speed of light, written as the symbol c , in vacuum or in air is 3.0×10^8 m/s. When light enters a material (through an interface) its speed can increase or decrease. The speed changes depending on the refractive index of the material. If x represents the medium and v_x is the speed of light in the medium, then the refractive index of the material is

$$n_x = \frac{c}{v_x}$$

If the refractive index is equal to 1.0, then the speed of light in medium x is the same as it is in air and no bending of the light ray occurs. The larger the refractive index the slower the speed of light is in the medium. When light enters a medium with a refractive greater than 1.0 the path of the ray changes. The relationship that describes the light ray entering a medium from one medium to another is called Snell's law:

$$n_1 \sin(i) = n_2 \sin(r)$$

i is angle of the incident (incoming) ray, r is the angle of the refracted ray and n_1 and n_2 are the refractive indices for the two medium the light ray travels in. The diagram below illustrates Snell's law for a light ray entering a glass medium from air.



The light ray travels in a path towards the air-glass interface at an incident angle i with respect to the normal (line perpendicular to glass surface). The path of the ray is altered due to the refractive index of the material and the refracted angle is r . Once the ray reaches the end of the glass medium the ray's path will refract again so as to be parallel to its path before entering the glass. The angle of the outgoing ray is the same as the angle of the incoming ray, i .

QUESTION 1

Light travels from air into flint glass with an angle of incidence of 30° . If the refractive index of glass is 1.66 and that of air is 1.00, what is the angle of the refracted ray?

⇒ Start by writing down Snell's law

$$n_1 \sin(i) = n_2 \sin(r)$$

⇒ Rearrange the equation

$$\sin(r) = \frac{n_1}{n_2} \sin(i)$$

⇒ To make r the subject, take the inverse sine

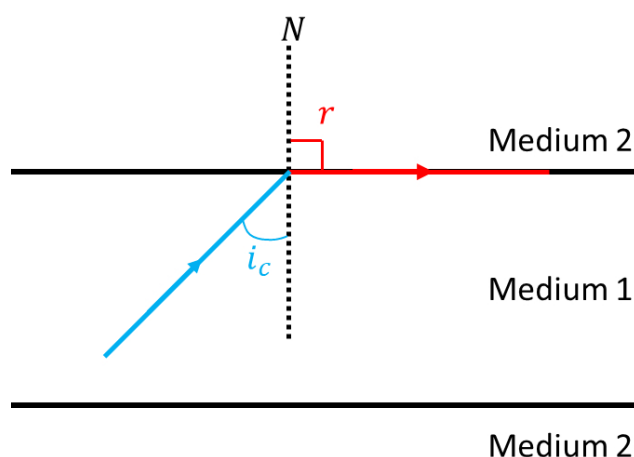
$$r = \sin^{-1}\left(\frac{n_1}{n_2} \sin(i)\right)$$

⇒ Putting in all the numbers for the refracted indices and incident angle, the refracted angle is

$$\begin{aligned} r &= \sin^{-1}\left(\frac{1.00}{1.66} \sin(30^\circ)\right) \\ &= 17.5^\circ \end{aligned}$$

CONTENT – TOTAL INTERNAL REFLECTION

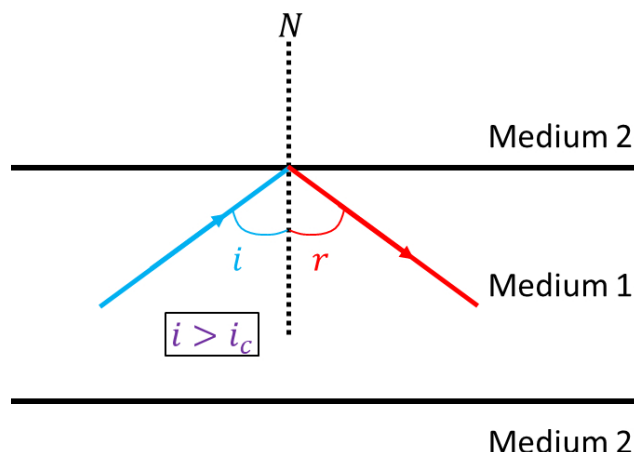
When light travels from an optically denser material to one less dense, (as, for example, from glass to air) the refracted ray is bent away from the normal to the surface. As the angle of incidence increases a situation will be reached where the refracted ray lies along the surface between the two materials (i.e. the refracted angle is $i_r = 90^\circ$). This angle of incidence is called the critical angle i_c . Below is a diagram illustrating the critical angle of the incoming ray at an interface.



Applying Snell's law to the situation above, the right-hand side of the equation becomes $n_2 \sin(90^\circ) = n_2$. If medium 2 is air the refractive index is equal to $n_2 = 1.00$. Thus, the sine of the critical angle is

$$\sin(i_c) = \frac{1}{n_x}$$

To determine the critical angle, we take the inverse of the sine of the right-hand side. At angles greater than the critical angle all the light is reflected, so no light escapes the optically denser material. This phenomenon is called total internal reflection (TIR). As illustrated below, any such ray obeys the Law of Reflection with the angle of incidence i equal to the angle of reflection r .



QUESTION 2

Light travelling inside crown glass with a refractive index of $n = 1.50$ approaches the glass-air interface. What is the minimum angle of incidence needed for the light to be totally reflected (i.e. critical angle i_c)?

⇒ We first write down the equation for the critical angle

$$\sin(i_c) = \frac{1}{n_x}$$

⇒ Rearranging to make the critical angle the subject

$$i_c = \sin^{-1}\left(\frac{1}{n_x}\right)$$

⇒ Substituting the refractive index of glass is

$$i_c = \sin^{-1}\left(\frac{1}{1.50}\right) \\ = 41.8^\circ$$

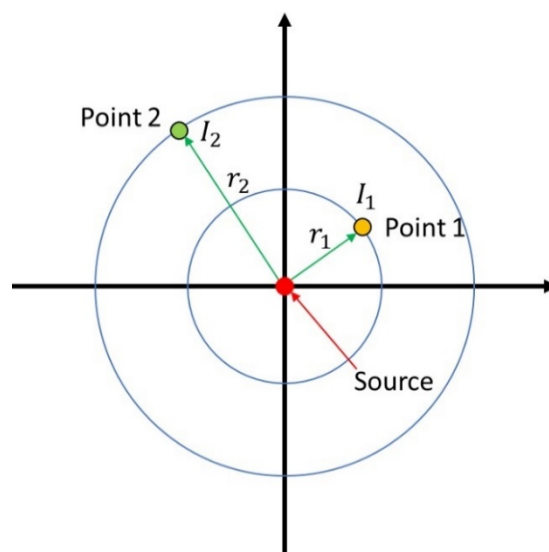
⇒ Therefore, the angle of incidence needs to be at least 41.8° for total internal reflection to occur.

CONTENT – LIGHT INTENSITY

As light travels in space, the intensity (power per unit area) of the light will gradually decrease. Thus, the light intensity is greater the closer you are to the light source. This is demonstrated in real life when you see an oncoming car at night the headlights are very faint. As the car comes closer to you, the headlights become very bright. The intensity, I , is inversely proportional to the square of the distance, r^2 , and this relationship is called the inverse-square law

$$I \propto \frac{1}{r^2}$$

The reason for the relationship is that the light is spread out over the surface of an expanding sphere. Following the inverse-square law, the intensities of light at two points, r_1 and r_2 from a point source at the origin, O , is given by



$$I_1 r_1^2 = I_2 r_2^2$$

The intensity multiplied by the distance squared must be equal at the two points because the light intensity at any point cannot be greater than the intensity at the source (i.e. conservation). Thus, if the distance r_2 is greater than r_1 the intensity I_2 must be smaller than I_1 to compensate.

Question 3

If the intensity of light at 0.5 m from a source is 100 W/m², what is the intensity of light at 3.0 m away from the source?

⇒ The inverse-square law that the relationship of the intensities at two points is

$$I_1 r_1^2 = I_2 r_2^2$$

⇒ We want to find the intensity at point 2, thus rearrange to make I_2 the subject

$$I_2 = \frac{r_1^2}{r_2^2} I_1$$

⇒ Substituting the known values, we get an intensity of

$$\begin{aligned} I_2 &= \frac{(0.5 \text{ m})^2}{(3.0 \text{ m})^2} \times 100 \text{ W/m}^2 \\ &= 2.78 \text{ W/m}^2 \end{aligned}$$

QUESTION 4 - Internet RESEARCH

Explain how the phenomenon of total internal reflection is used in fibre optics technology?

QUESTION 5 - Internet RESEARCH

How is the inverse-square law of light used in photography?

TEMPERATURE

CONTENT – TEMPERATURE AND KINETIC ENERGY

What is temperature? In our everyday life experience temperature is a measure of the degree of 'hotness' or 'coldness' of an object. The term 'heat' refers to energy (or more specifically thermal energy) and is related to temperature. The SI unit of heat or thermal energy is Joule (J). The temperature of an object can be increased by adding thermal energy to it, and it can also be decreased by removing thermal energy from the object. However, heat and temperature are not the same quantities. The temperature of an object determines the *average* kinetic energy of all particles (i.e. atoms and molecules) in the object. The higher the temperature, the larger the kinetic energy (particles on average move faster). If two objects are in thermal contact, thermal energy will be transferred from the object at the higher temperature to the object at the lower temperature.

QUESTION 1

Consider heating two beakers of water using a Bunsen burner – keeping all variables the same and simply changing the amount of water being heated. If the amount of water in Beaker A is 50 mL and in Beaker B 100 mL, which beaker will have a higher temperature after 1 minute?

CONTENT – TEMPERATURE SCALES

If temperature is an indicator of the average kinetic energy of particles in an object, how then should we report the value of temperature? Would a news outlet report on the temperature in Joules? Most likely not because it is not something that is humanly relatable. Thus, temperature scales were invented for convenience. For historical reasons, there are three temperature scales available used for measuring temperature – Fahrenheit, Celsius and Kelvin. The SI unit of temperature is Kelvin (K), and at 0 K all the particles in an object stop moving (zero thermal energy).

One or more fixed points are required for these three temperature scales. In the SI system, the triple boiling point of water is used as the internationally accepted fixed point; the temperature scale is defined by assigning the value of 273.15 K to this triple point.

The table below shows the **lower** (melting point of ice) and **upper** (boiling point of water) points of the three scales. For the Celsius and the Fahrenheit scales, they are fixed points.

Temperature scale	Lower fixed point	Upper fixed point
Fahrenheit	32 °F	212 °F
Celsius	0 °C	100 °C
Kelvin	273.15 K	373.15 K

The temperature measured in Celsius can be converted to the Kelvin scale adding 273.15 K to each reading on the Celsius scale. Fahrenheit measurement can be converted to Celsius by $T(^{\circ}\text{C}) = (T(^{\circ}\text{F}) - 32) \times \frac{5}{9}$.

QUESTION 2

Convert the following temperatures from:

Celsius to Kelvin:

- a) 26 °C
- b) 92 °C

Fahrenheit to Celsius:

- c) 85 °F
- d) 54 °F

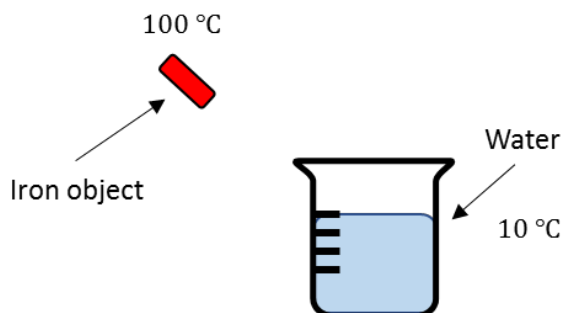
QUESTION 3 INTERNET RESEARCH

When was the first thermometer made?

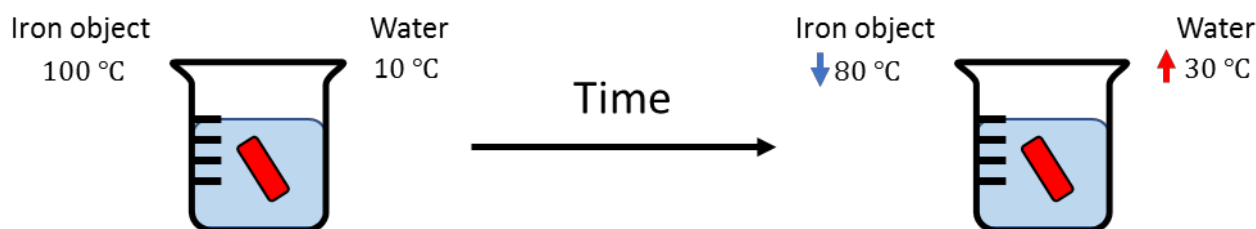
THERMAL EQUILIBRIUM

CONCEPT

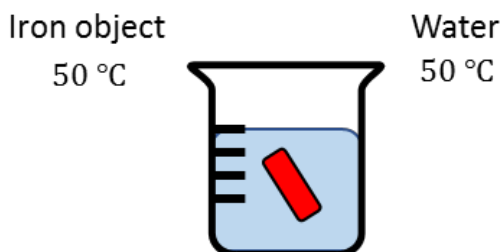
Two objects are said to be in thermal equilibrium if they are in contact with each other, and no heat transfer occurs. To understand this, consider a hot iron object placed in a container of cold water illustrated in the diagram below.



The iron has a higher temperature than the temperature of the cold water. When the iron object is dipped into the container (thermal contact) heat transfer occurs. Thermal energy is passed from the iron to the cold water. The iron object slowly loses heat while the cold water gains heat.



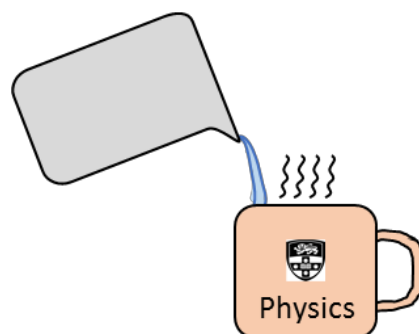
The heat transfer stops when both the iron object and water reaches the same temperature.



When this happens, the iron object and water are said to be in a state of thermal equilibrium.

EXAMPLE – REAL WORLD

The example above can be applied to a common real-world example: your coffee mug. When you pour hot water into the coffee mug, the hot water transfers energy (heat) to the mug. As a result, the temperature of the mug will increase. The temperature of the hot water will continue to decrease as the mug gets hotter. After some time, the temperature of the water and mug will be the same. Thus the two are in thermal equilibrium.



SPECIFIC HEAT CAPACITY

CONTENT

The previous two topics deal with heat or thermal energy and the difference between temperature and internal energy (the kinetic energy of particles within an object). The symbol for thermal energy is Q and has units of joules (J). A change in temperature results in a change in thermal energy. Writing the mass of an object as m , the initial and final temperatures of the object T_1 and T_2 and the resulting change in thermal energy as Q , we have the equation

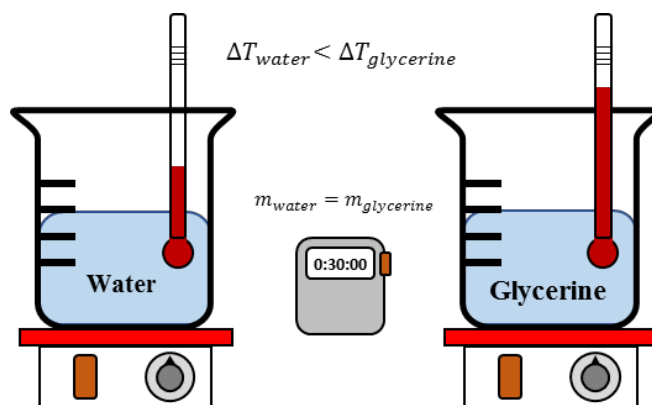
$$Q = mc(T_2 - T_1)$$

$$Q = mc\Delta T$$

In this equation, c is a property of matter called the specific heat capacity. The specific heat capacity is defined as the amount of thermal energy needed to raise the temperature of a 1 kg object by 1 °C. The SI unit for specific heat is J/(kg.°C). Different materials will behave differently to a change in temperature and is described by the value of the specific heat capacity. The change in temperature ΔT can either be positive or negative depending on whether thermal energy is entering or leaving the body.

EXAMPLE - LABORATORY

Consider equal masses of 1 kg of water and glycerine in separate beakers, both initially at the same temperature T_1 .



As both beakers are heated the temperatures as shown by the thermometers will increase. However, the increase shown on the thermometer in the glycerine will be greater than that shown on the thermometer in the water. This is because the specific heat of glycerine is less than that of water ($c_{\text{water}} = 4184 \text{ J/(kg.°C)}$ and $c_{\text{glycerine}} = 2430 \text{ J/(kg.°C)}$).

QUESTION 1 - INTERNET RESEARCH

Track down and compare the specific heat capacities of common substances – water, alcohol, ice, aluminium, iron, copper, gold, mercury, wood and steel.

QUESTION 2

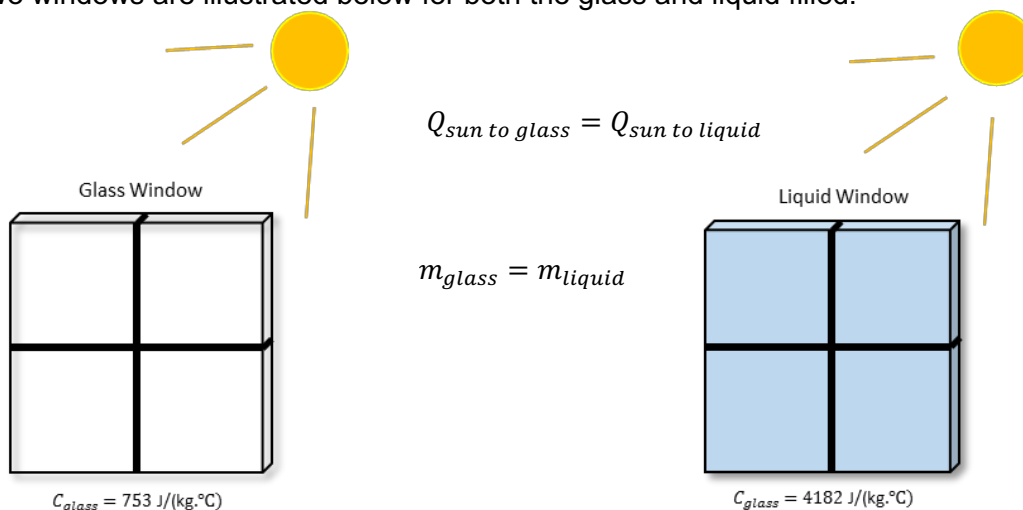
If we take 10 g of water and add 700 J by heating it, what will be the rise in its temperature? (Remember that the SI unit of mass is kg.)



EXAMPLE - REAL WORLD

In an attempt to reduce electric bills for homes, some innovators have developed liquid filled windows to cool houses in summer and help keep them warm in winter explain how a water-filled window can cool the house in summer but the opposite in winter. Is it better to use alcohol than water for the liquid instead? The specific heat of glass is $c_{\text{glass}} = 753 \text{ J/(kg}\cdot^{\circ}\text{C)}$, water is $c_{\text{water}} = 4184 \text{ J/(kg}\cdot^{\circ}\text{C)}$ and alcohol is $c_{\text{alcohol}} = 2440 \text{ J/(kg}\cdot^{\circ}\text{C)}$.

⇒ The two windows are illustrated below for both the glass and liquid filled.



⇒ Both windows should receive the same amount of thermal energy from the sun. Starting with the equation

$$Q = mc\Delta T$$

⇒ Rearrange to get the change in temperature

$$\Delta T = \frac{Q}{mc}$$

⇒ The change in temperatures for the two windows then becomes

$$\Delta T_{\text{glass}} = \frac{\text{Glass } Q}{m \times 753}$$

$$\Delta T_{\text{water}} = \frac{\text{Water-filled } Q}{m \times 4184}$$

$$\Delta T_{\text{glass}} > \Delta T_{\text{water}}$$

Since the specific heat capacity of glass is about 5x smaller than water

⇒ Therefore, the liquid filled window will have a change in temperature about five times smaller than the glass window.

⇒ If alcohol is used instead of water then the change in temperature is:

$$\Delta T_{\text{alcohol}} = \frac{\text{Alcohol } Q}{m \times 2440}$$

$$\Delta T_{\text{water}} = \frac{\text{Water-filled } Q}{m \times 4184}$$

$$\Delta T_{\text{alcohol}} > \Delta T_{\text{water}}$$

Since the specific heat capacity of alcohol is about 2x smaller than water

⇒ If we were to use alcohol to fill the window, the change in temperature would be about twice the change for the water-filled window. Therefore, it would be better to use water.

Investigation: Thermal Conductivity

INTRODUCTION

For understanding the concept of thermal conductivity, let's take an example - consider walking barefoot on a tiled floor and on a carpet. Which surface feels 'colder' to walk on? Walking on a tiled floor 'feels' colder than walking on a carpet. The differences in sensations are to do with the composition of both the tiled floor and the carpet. The tiled floor is made up of a different material than the carpet; hence the rate at which heat transfer occurs is not the same for both these surfaces. When heat transfer occurs between two objects that are in contact, we refer to this transfer of heat as *thermal conduction*.

What really happens when you stand barefoot on a tiled floor in winter? Think about the molecules in the two bodies that are in contact – what is their kinetic energy?

In this experiment, we will model and predict quantitatively energy transfer from hot objects by the process of thermal conductivity. Before carrying out the experiment, undertake an internet research activity to find the thermal conductivity values for a range of metals. What do you observe?

1. QUESTIONING AND PREDICTING

Think about the following questions to formulate an aim for the investigation that you will undertake:

1. What factors does thermal conductivity depend on?
2. Will heat transfer occur at the same rate for two substances made up of the same material, but of different thickness?
3. Do you think all substances (for example metals) conduct heat at the same rate?

HYPOTHESIS

Thermal conductivity varies/does not vary for different metals.
Different metals conduct heat at (different/same) rates.

2. PLANNING INVESTIGATION

This investigation has been planned for you.

In this experiment, we will investigate if different metals conduct heat at different or same rates. To this end, we will use a range of metal wires of same length and thickness, a Bunsen burner, wax blobs and paper clips of equal size, heat proof mat, matches, stop watch, tripod and safety glasses.

Before undertaking any science experiment, it is important to do a risk assessment to assess any risks and devise strategies to mitigate those risks. Complete the table below.

Potential risks	Strategies to mitigate the risks

The steps below outline the procedure for carrying out the experiment.

1. Place the Bunsen burner on a heat proof mat and ensure that the air hole of the Bunsen burner is closed (*Recall* the difference between the safety flame and blue flame of a Bunsen burner.)
2. Place a tripod stand on top of the Bunsen burner.
3. Arrange a range of metal wires on top of the tripod stand ensuring that they all get exposed to the heat from the Bunsen burner in the same way, when we'll light the Bunsen burner in Step 5.
4. Place a wax blob at the end of each wire and attach a paper clip to each blob of wax.
5. Have a stop watch ready (you can use one on your phone) and light the Bunsen burner.
6. Record the time taken for the wax to melt resulting in the paper clip to fall from each wire. When will you stop recording the time? When the Wax starts melting or when the paper clip falls?

3. CONDUCTING INVESTIGATION

Record your results in the table below.

Metals	Time (s)

Is your experiment 'reliable'? Why/why not?

Can you think of any errors that likely affected your results?

How can you improve the experiment?

4. PROCESSING AND ANALYSING

Plot the results on a graph paper or in Microsoft Excel. Will you make a line graph or a column/ bar graph? What graph would best represent your results?

What were the independent, dependent and control variables in your experiment?

Is your experiment valid?

5. PROBLEM SOLVING

Refer to the worksheet on Energy Transfer, and work with the following equation:

$$P = \frac{Q}{t} = \frac{kA\Delta T}{d} = \frac{kA(T_2 - T_1)}{d}$$

What do the different components of the thermal conduction equation mean?

As an extension to this experiment, plan another investigation to test if same metals with varying thickness conduct heat at the same time. What could be your hypothesis?

Thermal conductivity varies/does not vary for same metals with varying thickness
Same metals with varying thickness conduct heat at (different/same) rates.

6. CONCLUSIONS

Fill in the blanks with one or many words.

Different metals conduct heat at _____ rates.

Thermal conductivity is _____ for different metals.

Thermal conductivity _____ for same metals with varying thickness.

Same metals with varying thickness conduct heat at _____ rates.

Investigation: Latent Heat of Vaporisation

INTRODUCTION

When we want to increase the temperature of a substance, we need to supply heat. The amount of heat energy it takes to raise the temperature by one degree depends on the heat capacity.

Something different happens when we reach a phase change. If you take ice out of the freezer, it warms up until it starts to melt – and then stays at 0°C until it finishes melting! Does this mean that heat has stopped transferring to the ice?

In this experiment, we will analyse the heat transfer involved when water boils. We will be looking at what happens to the temperature of the water, and how much energy it takes to convert water into steam.

1. QUESTIONING AND PREDICTING

Let us think about the aim of this investigation.

1. How does the temperature of water change when it is near or at its boiling point?
2. How much energy do we have to supply to boil a certain volume of water?

HYPOTHESIS

The temperature will _____ (increase/remain constant) once the water reaches its boiling point.

The amount of energy needed to boil a given volume of water is a constant.

2. PLANNING INVESTIGATION

This investigation has been planned for you.

Each group will be given a Bunsen burner, a stopwatch, a beaker of water and a thermometer. There will be a weighing scale available for use. First, we must determine the rate at which the Bunsen burner supplies heat to the water.

1. Measure and record the temperature of the water before heating.
2. Measure the initial mass of the water. (Hint: find the mass of an empty beaker and subtract this from your measured value.)
3. Heat the water over the burner for one minute. Ensure it does not begin to boil at this stage.
4. Remove the beaker from the heat source. Stir gently to ensure the temperature is consistent and record the increased temperature.



5. Use the equation $\Delta Q = mc\Delta T$ to determine how much heat was transferred to the water in one minute. Use the value $c = 4.18 \text{ kJ/kg/}^\circ\text{C}$.

You can now assume the burner provides heat to the water at this constant rate. In the second phase of the experiment, we will examine how much heat is required to boil a certain mass of water.

6. Continue heating the water until it reaches boiling point at 100°C . Start timing on the stopwatch.
7. Record the temperature every 20 seconds and observe any trends.
8. After approximately three minutes, remove the beaker from the heat source. Determine the mass of water which remains in the beaker.
9. Use your value for the rate of heating (from step 5) to determine how much heat was required to boil off the difference in mass.
10. Using the equation $Q = mL$, calculate the latent heat of vaporisation L for water in kJ/kg .

3. CONDUCTING INVESTIGATION

Each group should fill in the following as you go:

Initial mass of water: _____

Initial temperature of water: _____

Temperature of water after heating: _____

Record the observations of temperature while boiling the water:

Final mass of water: _____

Final mass – initial mass: _____

Did you make any changes to the method? What safety considerations were important? Did you have some 'smart' ways of doing the investigation?

4. PROCESSING AND ANALYSING

Before we can work out how much energy was supplied to boil the water, we have to know the approximate rate at which heat is transferred from the Bunsen burner to the water.

If a mass m of water is heated by an amount ΔT , then we can calculate how much heat was supplied in one minute, using the known value of the heat capacity of water $c = 4.18 \text{ kJ/kg/}^\circ\text{C}$.

The equation to use is $\Delta Q = mc\Delta T$.

Therefore Q kilojoules are supplied per minute. When you move on to boiling the water, you can determine the heat transferred to the water by multiplying this factor by the number of minutes during which it was boiling.

To determine the latent heat of vaporisation, divide this amount of heat by the mass of water lost to find the answer in kJ/kg. Note that sometimes this quantity is expressed in J/kg or J/g.

5. PROBLEM SOLVING

When a substance is undergoing a phase change, its temperature remains constant. However, in the case of melting or evaporation, heat energy must still be entering the system. The latent heat L of vaporisation is a quantity which describes how much energy is needed to evaporate a kilogram of water once it has reached its boiling point.

There is also a corresponding quality, the latent heat of fusion, which relates to the solid \rightarrow liquid phase transition.

For water, the latent heat of **fusion** is 334 kJ/kg.

The latent heat of **vaporisation** is 2,230 kJ/kg. How does this compare to the answer you found?

6. CONCLUSIONS

During a phase change from liquid to gas, the temperature of a substance _____.

The experimentally measured latent heat of vaporisation of water is _____.

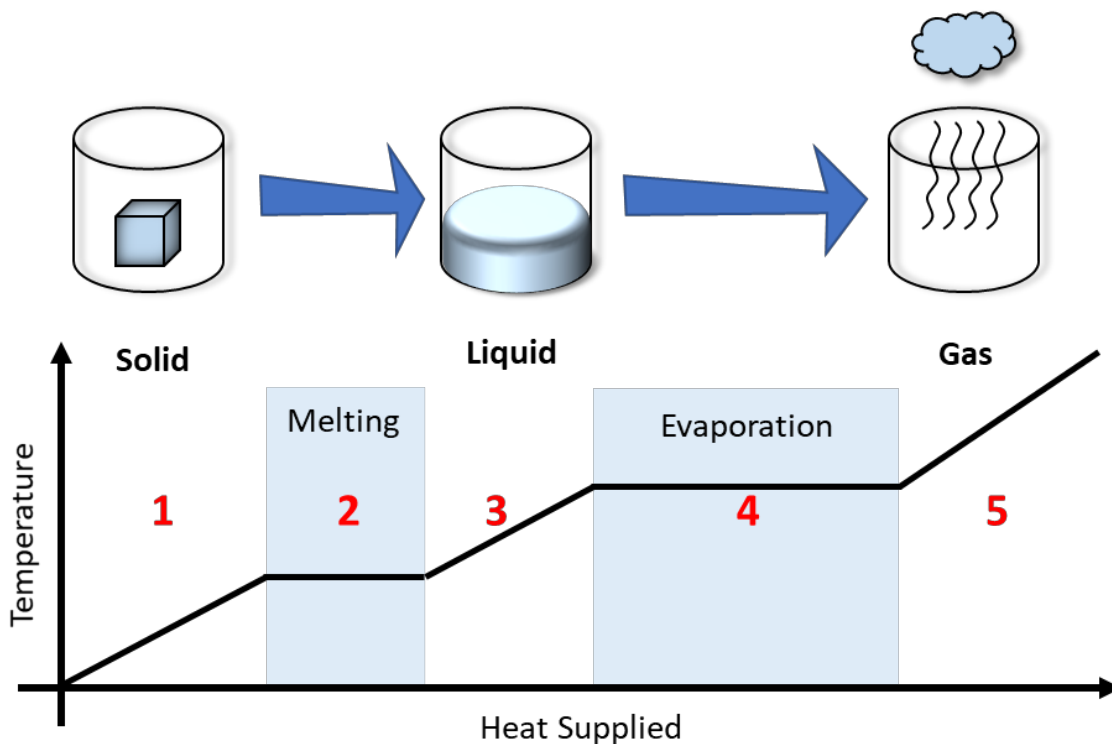
LATENT HEAT

CONTENT

Matter exists in different states, and the three common states are solids, liquids and gases. Matter in one state can transition to another state; a phenomenon called *phase change*. When a substance changes phase, thermal energy is absorbed by the substance from its surroundings. The temperature of the substance, however, remains constant during the phase change. The heat or thermal energy per unit mass is called the latent heat of the phase change. For historical reasons the latent heat for *melting* (solid → liquid) is called the **latent heat of fusion** and *evaporation* (liquid → gas) is called **latent heat of vaporisation**. The thermal energy required or released due to phase change is given by

$$Q = mL$$

where Q is the thermal energy, m is the mass and L is the latent heat. The units of latent heat are joule per kilogram (J/kg). Below is a diagram showing the different processes for water.



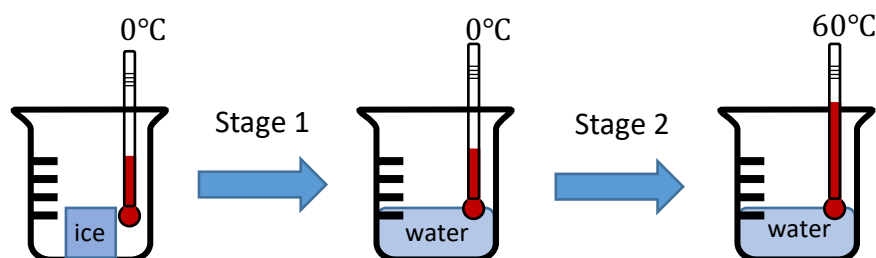
Following the diagram above:

1. At low temperatures, water is in the solid form we call 'ice'. Adding thermal energy to ice will increase the temperature of the ice.
2. At some stage, the ice will **melt** into liquid (i.e. undergo a phase change) but the temperature will remain the same during this process.
3. With further thermal energy added, the temperature of the liquid will rise again.
4. At a much higher temperature, the liquid will start to **evaporate** into gas (a second phase change). The temperature will also not change during this process similar to melting.
5. Once the phase change is complete, the temperature of the gas will increase.

WORKED EXAMPLE - ICE TO WATER

How much energy, as heat flow, is required to change 0.40 kg of ice at 0°C into the water at 60°C? The latent heat of fusion of water is 335,000 J/kg, and the specific heat is 4184 J/(kg.°C).

⇒ Let's break down the question into stages. The ice melts into the water without any change in temperature (a phase change). After this, the water temperature is increased from 0°C to 60°C.



⇒ Let's now write down all the information we are given or know

<ul style="list-style-type: none"> • Energy due to phase change $Q = mL$ • Energy from heating $Q = mC(T_f - T_i)$ • Mass of water and ice, $m_{ice} = m_{water} = 0.40 \text{ kg}$ 	<ul style="list-style-type: none"> • Temperature changes from $T_i = 0^\circ\text{C}$ to $T_f = 60^\circ\text{C}$ • Latent heat of fusion is $L = 335,000 \text{ J/kg}$ • Specific heat of water is $C = 4184 \text{ J/(kg.}^\circ\text{C)}$
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⇒ Starting with **stage 1** we find the amount of energy required to melt the ice.

Energy from melting the ice = mass of ice × latent heat of ice

$$\begin{aligned}
 Q_{ice} &= mL \\
 &= (0.40 \text{ kg}) \times (335,000 \text{ J/kg}) \\
 &= 0.134 \text{ MJ}
 \end{aligned}$$

⇒ Moving to **stage 2** we find the thermal energy required to raise the temperature of the water from 0°C to 60°C

Energy from heating water = mass of water × specific heat of water × temperature change

$$\begin{aligned}
 Q_{water} &= mC(T_f - T_i) \\
 &= (0.40 \text{ kg}) \times (4184 \text{ J/(kg.}^\circ\text{C)}) \times (60^\circ\text{C} - 0^\circ\text{C}) \\
 &= 0.1004 \text{ MJ}
 \end{aligned}$$

⇒ Finally, the total heat flow required is the sum of the energies from the two stages.

Total heat flow required = Energy of ice melting + Energy of heating water

$$\begin{aligned}
 Q_{Total} &= Q_{ice} + Q_{water} \\
 &= 0.134 \text{ MJ} + 0.1004 \text{ MJ} \\
 &= 0.23 \text{ MJ}
 \end{aligned}$$

ENERGY TRANSFER

CONTENT

When thermal energy is absorbed or released from an object a change in temperature results. When an object receives thermal energy, the energy is obtained from a source. However, the thermal energy must be transferred from the source to the object. This process is called '**energy transfer**', and there are three mechanisms in which energy is transferred from one object to another.

CONDUCTION

In this mechanism, energy is transferred through direct/physical contact. For example, if you touch a hot stove, thermal energy will be transferred from the hot surface to your hand. We can determine the rate of **thermal conduction** (labelled as P , with units joules/second, or watts) by using the equation

$$P = \frac{Q}{t} = \frac{kA\Delta T}{d} = \frac{kA(T_1 - T_2)}{d}$$

where Q is thermal energy, t is the time, k is thermal conductivity of a material, A is the cross-sectional area where heat transfer occurs, d is the thickness and ΔT is the difference in temperature between two objects. Each material will have a different thermal conductivity value (e.g. copper has a high thermal conductivity in comparison to wood), and the unit is watts per meter kelvin $W/(m.K)$. From the equation above the thermal conduction P is inversely proportional to the thickness d . This means that a higher thermal conductivity will result in a higher rate of conduction and larger thickness will result in a lower rate of conduction.



Figure 1.

Think: Taking the above equation into account, explain why thick clothing keeps your body warmer than thin clothing (made out of the same material) in winters?

CONVECTION

Higher temperatures will translate to higher kinetic energy. Using boiling water in a pot as an example (image on the right), water molecules that are closer to the bottom will have a higher kinetic energy. With a higher energy, the water molecules will move and collide with other molecules more often causing the molecules to be further apart. This results in a lower density and consequent rising through the rest of the liquid (like an inflatable ball released at the bottom of the pool). This process is called **convection currents** where the heated fluid moves away from the heat source, in turn carrying and transferring energy. The transfer of energy results in the increase of temperature of water in the pot.

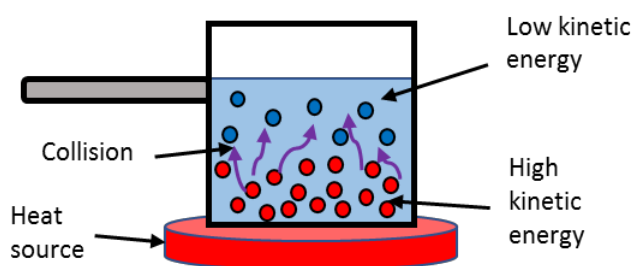


Figure 2.

RADIATION

We experience energy transfer due to radiation almost every day. When we stand outside in the sun, we feel 'warm' because the energy from the sun is transferred to your body. To discern the difference between thermal and other electromagnetic signals (like x-rays or gamma (γ)-rays) we refer to this as **thermal radiation**. For example, when a stove is on, we can feel 'heat' when we stand near it. Why? Because the stove is emitting infrared waves into the air and you just happened to be standing nearby. The infrared waves will cause the molecules in your body to move faster, which explains your body heating up.

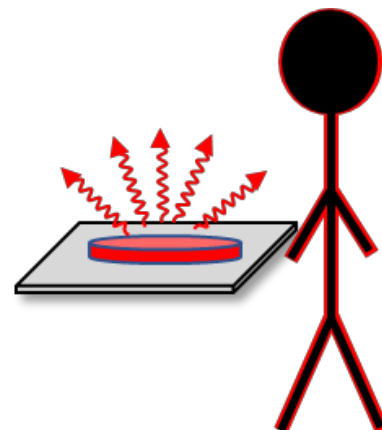
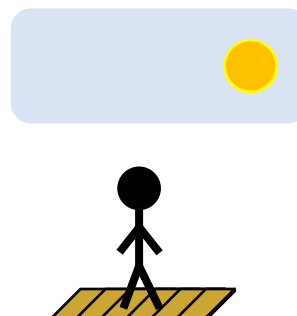
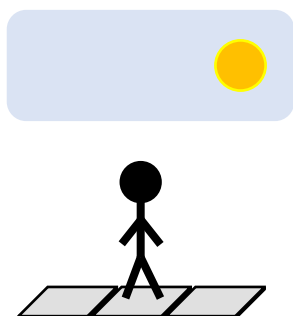


Figure 3.

EXAMPLE - REAL WORLD

On a hot sunny day on the beach, you walk barefoot on a concrete path. After some time your feet get hot, and you run to a wooden platform. On this surface, your feet are not as 'hot' as it was on the concrete surface. Why is it 'cooler' to stand on the wooden surface? What would happen if it is during a cold night instead? (The thermal conductivity for concrete is 1.7 W/(m. K) and wood is 0.17 W/(m. K))



⇒ For this question, we start with the thermal conductivity equation

$$P = \kappa \frac{A\Delta T}{d}$$

⇒ Concrete and wood have different thermal conductivity (i.e. κ)

$$\kappa_{\text{concrete}} = 1.70 \text{ W/(m. K)}$$

$$\kappa_{\text{wood}} = 0.17 \text{ W/(m. K)}$$

⇒ The thermal conductance from both surfaces are

$$P_{\text{concrete}} = 1.70 \times \frac{A\Delta T}{d}$$

$$P_{\text{wood}} = 0.17 \times \frac{A\Delta T}{d}$$

$$P_{\text{concrete}} \approx 10 \times P_{\text{wood}}$$

⇒ This means that the concrete surface absorbs heat from the sun about 10x faster than the wooden surface. Thus, the concrete surface is hotter than the wooden platform.

⇒ In a cold night the scenario becomes:



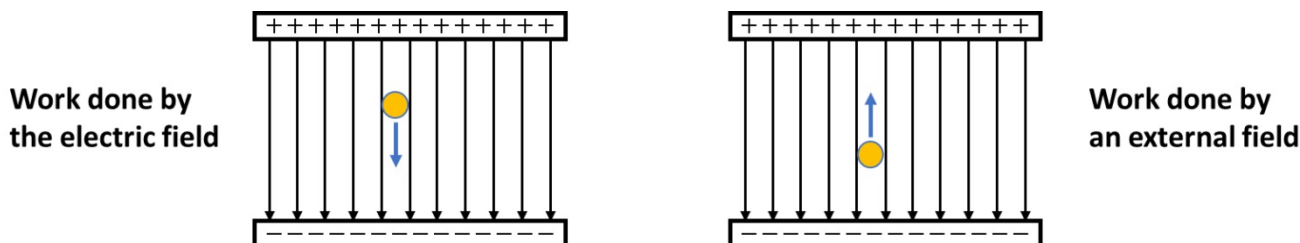
- ⇒ The reverse will happen, that is the concrete surface will feel 'colder' than the wooden surface. The concrete surface will release thermal energy 10x faster than the wooden surface. This is also because the thermal conductivity of concrete is 10x larger than wood.
- ⇒ In this example, we see that heat transfer goes both ways (absorbs or releases thermal energy) and we can feel this through direct contact with the surface of an object.



CHARGES IN AN ELECTRIC FIELD

CONTENT – POTENTIAL ENERGY AND WORK

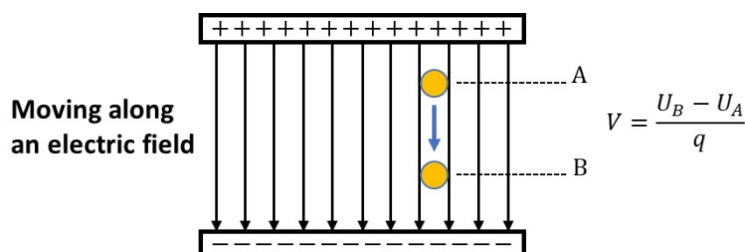
A charge in an electric field is analogous to an object in a gravitational field. An object positioned at higher gravitational potential energy will, if unrestrained, naturally move towards lower potential energy. In this situation, work is done by the gravitational force. If we move the object from a position of lower gravitational potential energy to higher potential energy, then work is done by the external force used to move it. Similarly, a charge in an electric field, will, if unrestrained, naturally move from a higher to lower electric potential energy. The reverse movement from a position of lower electrical potential energy will require an external force to do the work. This is illustrated in the diagram below for a charged particle in an electric field created by two conducting parallel plates. The arrows represent the electric field direction.



The potential difference between two points A and B in the electric field is defined as the amount of work done in moving a unit charge from A to B and is given by

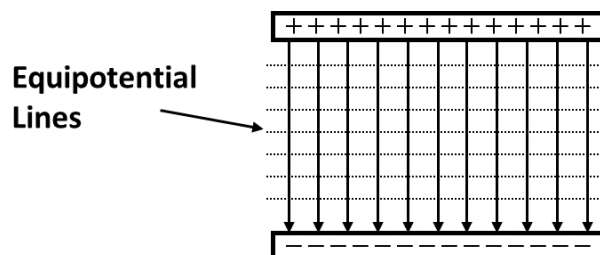
$$V = \frac{\Delta U}{q} = \frac{U_B - U_A}{q}$$

Where ΔU is the difference in potential energy between point A and B and q is the charge of the particle. V is the potential difference or voltage and is given in units of Volts (V) (This concept is important when we deal with electric circuits).



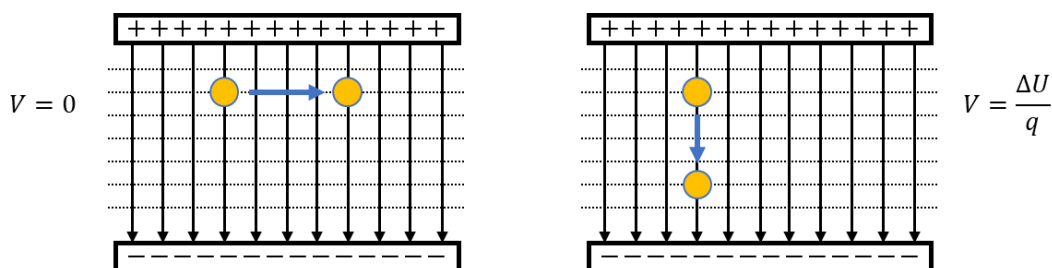
CONTENT – EQUIPOTENTIAL LINES

The electric potential can be represented graphically by drawing lines that are perpendicular to the electric field. These lines represent the points at which the electric potential energy is the same. Hence the name equipotential lines. The concept of equipotential lines in two dimensions can be extended to equipotential surfaces in three dimensions. For two parallel conducting plates, the equipotential surfaces are surfaces parallel to the plates.

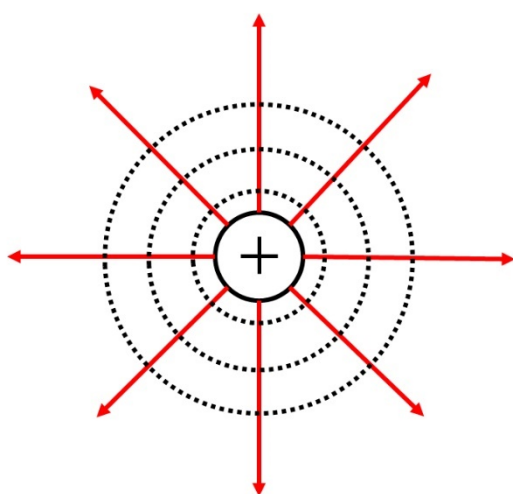




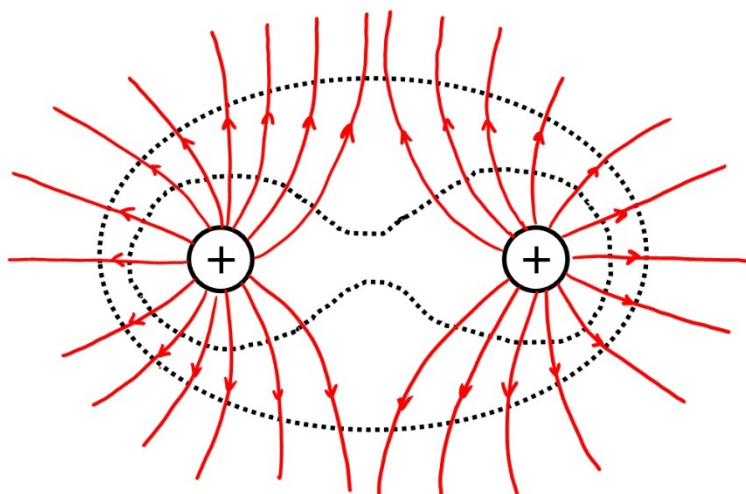
Since the electric potential along an equipotential line is the same, no work is done moving a charged particle along the equipotential line. However, moving the particle from one equipotential line to another will require work either by the electric field or an external force.



Below are two more examples of equipotential lines for a single positive charge and for two positive charges. Red lines represent the electric field lines while the dashed lines represent the equipotential lines.



Single Charge



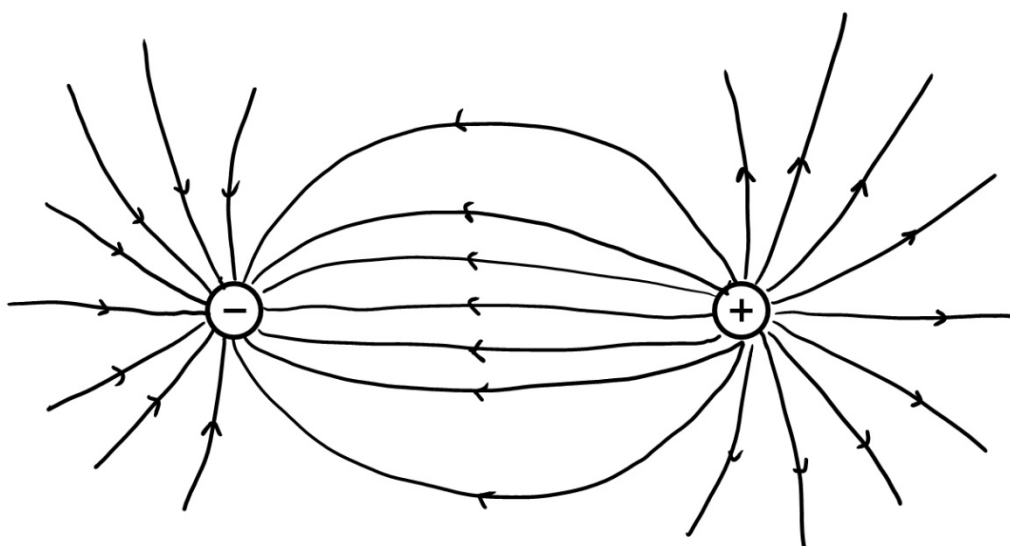
Double Charge

QUESTION 1

Calculate the amount of work done by an electric field (1.0 V/m) on a particle with a charge of 1.0 C that moves along an equipotential surface.

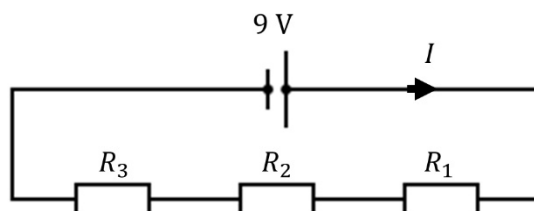
QUESTION 2 - PENCIL

Draw the equipotential lines for the positive and negative charges below.



WORKED EXAMPLE – SERIES CIRCUIT

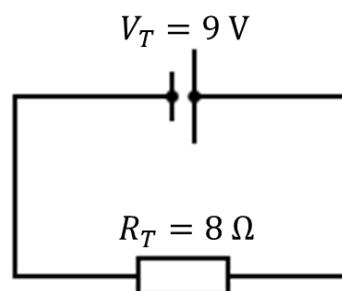
Find the voltage running through each resistor for the series circuit on the right. The resistance values are $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$ and $R_3 = 5\ \Omega$.



⇒ We have three resistors connected in series, and according to *Kirchoff's Current Law*, the current passing through each resistor is the same. Thus, if we know the total current is running through the circuit, we know the current running through each resistor.

⇒ We start by combining the three resistors connected in series into one equivalent resistor

$$\begin{aligned} R_T &= R_1 + R_2 + R_3 \\ &= 1\ \Omega + 2\ \Omega + 5\ \Omega \\ &= 8\ \Omega \end{aligned}$$



⇒ The total current running through the whole circuit is:

$$\begin{aligned} V &= IR \\ I_T &= \frac{V_T}{R_T} = \frac{9\ \text{V}}{8\ \Omega} = 1.125\ \text{A} \end{aligned}$$

⇒ Thus, the voltage across each resistor is:

$$\begin{aligned} V_{R_1} &= I_T R_1 \\ &= 1.13\ \text{A} \times 1\ \Omega \\ &= 1.125\ \text{V} \end{aligned}$$

$$\begin{aligned} V_{R_2} &= I_T R_2 \\ &= 1.13\ \text{A} \times 2\ \Omega \\ &= 2.250\ \text{V} \end{aligned}$$

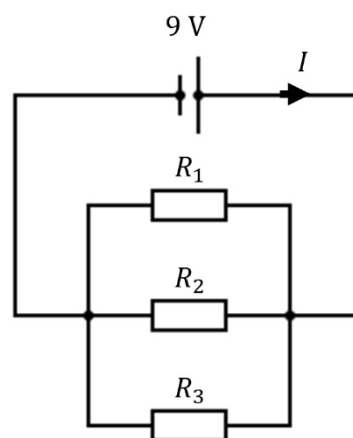
$$\begin{aligned} V_{R_1} &= I_T R_1 \\ &= 1.13\ \text{A} \times 5\ \Omega \\ &= 5.625\ \text{V} \end{aligned}$$

⇒ To check if we calculated the voltage correctly, make sure the sum of the voltages across each resistor is equal to 9.0 V (*Kirchoff's Voltage Law*).

$$\begin{aligned} V_T &= V_{R_1} + V_{R_2} + V_{R_3} \\ &= 1.125\ \text{V} + 2.250\ \text{V} + 5.625\ \text{V} \\ &= 9.0\ \text{V} \end{aligned}$$

WORKED EXAMPLE – PARALLEL CIRCUIT

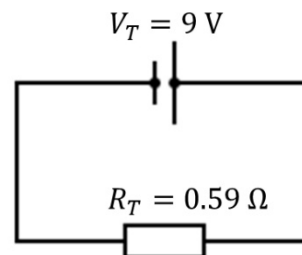
Find the current running through each resistor for the parallel circuit on the right. The resistance values are $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$ and $R_3 = 5\ \Omega$.



⇒ The three resistors are connected in parallel and the current entering and exiting the node of the resistors is the same. The voltage across each resistor is the same.

⇒ We combine the three resistors into one equivalent resistor

$$\begin{aligned}\frac{1}{R_T} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \\ &= \frac{1}{1\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{5\ \Omega} \\ &= \frac{17}{10\ \Omega}\end{aligned}\quad R_T = \frac{1}{17/10\ \Omega} = 0.59\ \Omega$$



⇒ The total current running through the whole circuit is:

$$V = IR$$

$$I_T = \frac{V_T}{R_T} = \frac{9\ \text{V}}{0.59\ \Omega} = 15.3\ \text{A}$$

⇒ The voltage running through each resistor is 9 V, and thus the current through each resistor is:

$$\begin{aligned}I_{R_1} &= \frac{V_T}{R_1} \\ &= \frac{9\ \text{V}}{1\ \Omega} \\ &= 9.0\ \text{A}\end{aligned}$$

$$\begin{aligned}I_{R_2} &= \frac{V_T}{R_2} \\ &= \frac{9\ \text{V}}{2\ \Omega} \\ &= 4.5\ \text{A}\end{aligned}$$

$$\begin{aligned}I_{R_3} &= \frac{V_T}{R_3} \\ &= \frac{9\ \text{V}}{5\ \Omega} \\ &= 1.8\ \text{A}\end{aligned}$$

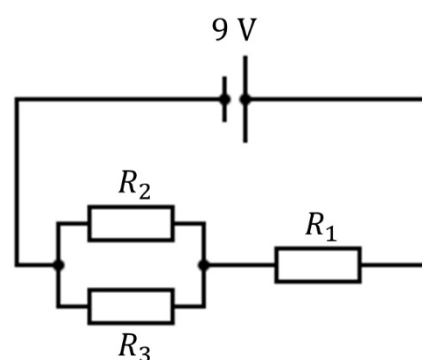
⇒ Check that the sum of the currents is equal to the total current:

$$\begin{aligned}I_T &= I_1 + I_2 + I_3 \\ &= 9.0\ \text{A} + 4.5\ \text{A} + 1.8\ \text{A} \\ &= 15.3\ \text{A}\end{aligned}$$

⇒ Observe that adding resistances (e.g. 1 Ω, 2 Ω and 5 Ω) in parallel decreases the total resistance (0.59 Ω)

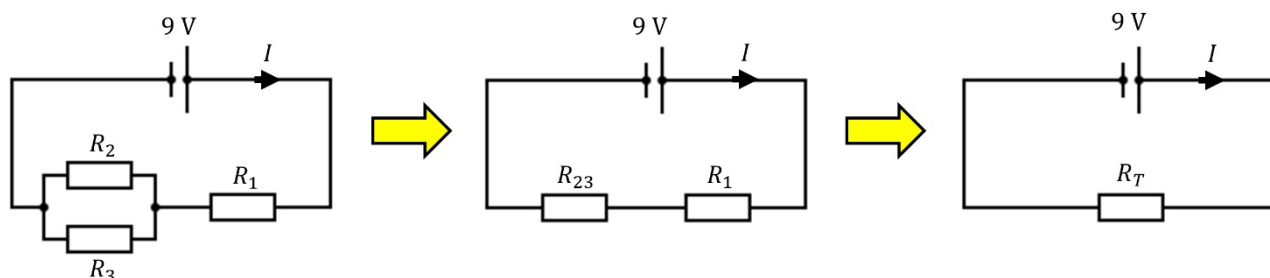
WORKED EXAMPLE – COMBINED SERIES AND PARALLEL CIRCUIT

Find the voltage and current running through each resistor for the series-parallel circuit on the right. The resistance values are $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$ and $R_3 = 5\ \Omega$.



⇒ For this circuit, the current flows through R_1 in series and then splits on a node to R_2 and R_3 in parallel. Much like solving the series and parallel only circuit we want to reduce the circuit into a one resistor circuit.

⇒ We first combine the resistors connected in parallel and then combine the resistors in series. Pictorially we perform the transformation below.





⇒ Below is the working out of combining the resistors as shown above

$$\begin{aligned}\frac{1}{R_{23}} &= \frac{1}{R_2} + \frac{1}{R_3} \\ &= \frac{1}{2\ \Omega} + \frac{1}{5\ \Omega} \\ &= \frac{7}{10\ \Omega}\end{aligned}$$

$$\begin{aligned}R_{23} &= \frac{1}{\frac{7}{10\ \Omega}} \\ &= 1.43\ \Omega\end{aligned}$$

$$\begin{aligned}R_T &= R_{23} + R_1 \\ &= 1.43\ \Omega + 1\ \Omega \\ &= 2.43\ \Omega\end{aligned}$$

⇒ The total current is:

$$\begin{aligned}V &= IR \\ I_T &= \frac{V_T}{R_T} = \frac{9.0\ \text{V}}{2.43\ \Omega} = 3.70\ \text{A}\end{aligned}$$

⇒ Once we have the total current, we work our way back and expand the circuit to get the current and voltages

⇒ The current is constant for resistors connected in series. Thus 3.70 A flows through both R_{23} and R_1 .

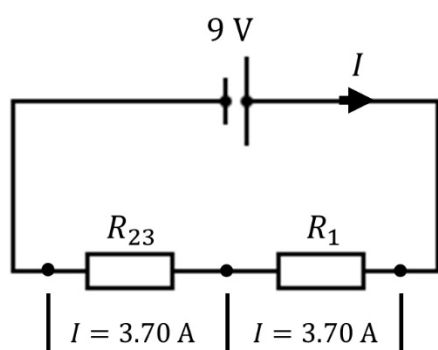
⇒ The voltage across R_{23} and R_1 is:

$$\begin{aligned}V_{R_{23}} &= I_T R_{23} \\ &= 3.70\ \text{A} \times 1.43\ \Omega \\ &= 5.30\ \text{V}\end{aligned}$$

$$\begin{aligned}V_{R_1} &= I_T R_1 \\ &= 3.70\ \text{A} \times 1\ \Omega \\ &= 3.70\ \text{V}\end{aligned}$$

⇒ Check sum of voltage is equal to 9.0 V:

$$\begin{aligned}V_T &= V_{R_{23}} + V_{R_1} \\ &= 5.30\ \text{V} + 3.70\ \text{V} \\ &= 9.0\ \text{V}\end{aligned}$$

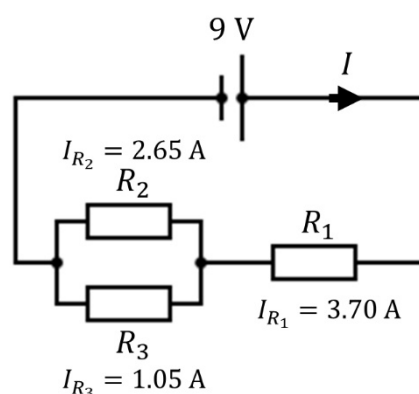


⇒ Now that we have the voltage running through R_{23} we can calculate the current through R_2 and R_3 .

$$\begin{aligned}I_{R_2} &= \frac{V_{R_{23}}}{R_2} = \frac{5.29\ \text{V}}{2\ \Omega} = 2.65\ \text{A} \\ I_{R_3} &= \frac{V_{R_{23}}}{R_3} = \frac{5.29\ \text{V}}{5\ \Omega} = 1.05\ \text{A}\end{aligned}$$

⇒ Check that the sum of the currents is equal to 3.70 A

$$\begin{aligned}I_{R_{23}} &= I_{R_2} + I_{R_3} \\ &= 2.65\ \text{A} + 1.05\ \text{A} \\ &= 3.70\ \text{A}\end{aligned}$$



⇒ Summary of the current and voltages:

Voltage (V)		Current (A)	
V_1	3.70	I_1	3.70
V_2	5.30	I_2	2.65
V_3	5.30	I_3	1.05
V_T	9.00	I_T	3.70

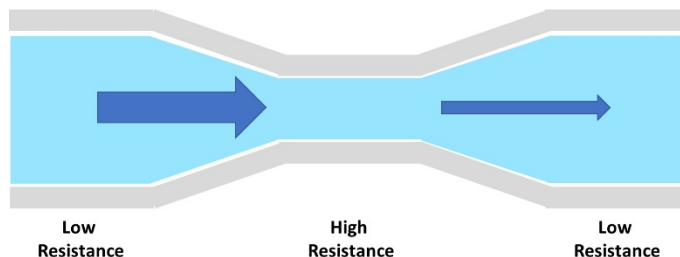
ELECTRIC CIRCUITS

CONTENT – OHM'S LAW

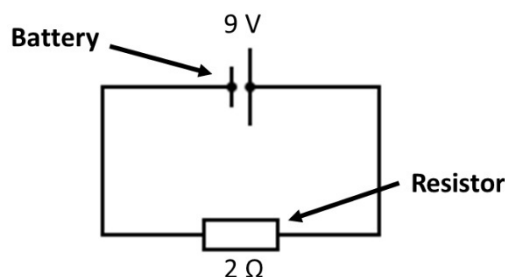
Ohm's law relates the voltage (i.e. potential difference V) with the current and resistance of an ideal conductor. The law states that

$$V = IR$$

where I is the current and R is the resistance of the conductor. The units for current and resistance is Ampere (A) and Ohm (Ω) respectively. The current is a measure of the flow of electrons in the circuit and is inversely proportional to the resistance. Hence, a large resistance will reduce the flow of electrons. As an analogy, this is similar to water flowing through a tube. A narrow tube will reduce the amount of water flowing through whilst larger tube will allow more water to flow through.



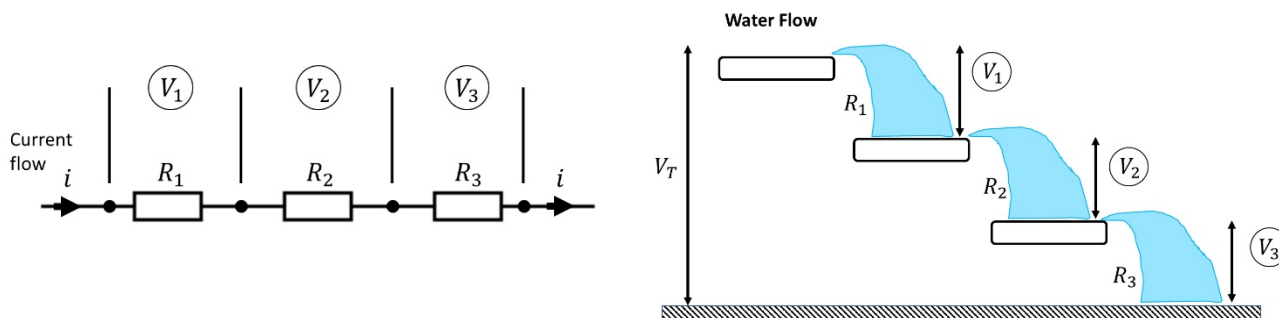
As a simple example, consider a one resistor circuit connected to a battery on the right. The battery provides a source of 9 V, and the resistance of the resistor is $2\ \Omega$. With the information we have, we can calculate the current flowing through the resistor using Ohm's law. The current flowing through the $2\ \Omega$ resistor is $I = V/R = 9\text{ V}/2\ \Omega = 4.5\text{ A}$. If the resistance is doubled to $4\ \Omega$ the current is reduced to 2.25 A.



It is important to note that resistors can either be ohmic or non-ohmic. Ohmic resistors have a constant resistance, hence in the equation above, if a resistor is an ohmic resistor with a constant resistance, then the graph of velocity against time is a straight line. In the case of non-ohmic resistors, where resistance in a resistor varies with, say current, then the graph of velocity against time is not a straight line. In the case of latter, the resistor is said not to obey Ohm's Law.

CONTENT – SERIES CIRCUITS

For a single resistor circuit like the circuit above applying Ohm's law is trivial. However, when more than one resistor is connected in either series or parallel, then Ohm's law cannot be applied the same way. For resistors connected in series (i.e. connected next to each other) the current flowing through each resistor is the same, but the voltage is divided amongst the resistors. For the circuit below with an incoming voltage of V_T , the voltage drop after passing through R_1 is $V_T - V_1$. Then the voltage drop after passing through R_2 is $V_T - V_1 - V_2$ and similarly for R_3 . Thus, after passing through each resistor the potential of the current flow decreases. If all resistors in series have same resistance, then the voltage drop across each resistor is the same. However, in the case of resistors with different resistance, the resistor with a greater resistance will result in the greatest voltage drop.



We can also use the analogy of water flowing from a higher to lower potential energy. Each drop in potential energy represents the electric current flowing through a resistor. When the water flows horizontally, there is no change in potential energy (like an electric current flowing through the wire). The sum of each potential energy drop is equal to the total potential energy the water is at initially. In electric circuits, this is called the **Kirchoff's Voltage Law (KVL)**, which states that the total potential difference in a closed circuit is equal to zero (i.e. conservation of energy).



$$V_{\text{Series}} = V_T = V_1 + V_2 + \dots + V_n$$

The total resistance of resistors connected in series can be calculated by summing up the resistance value of each resistor.

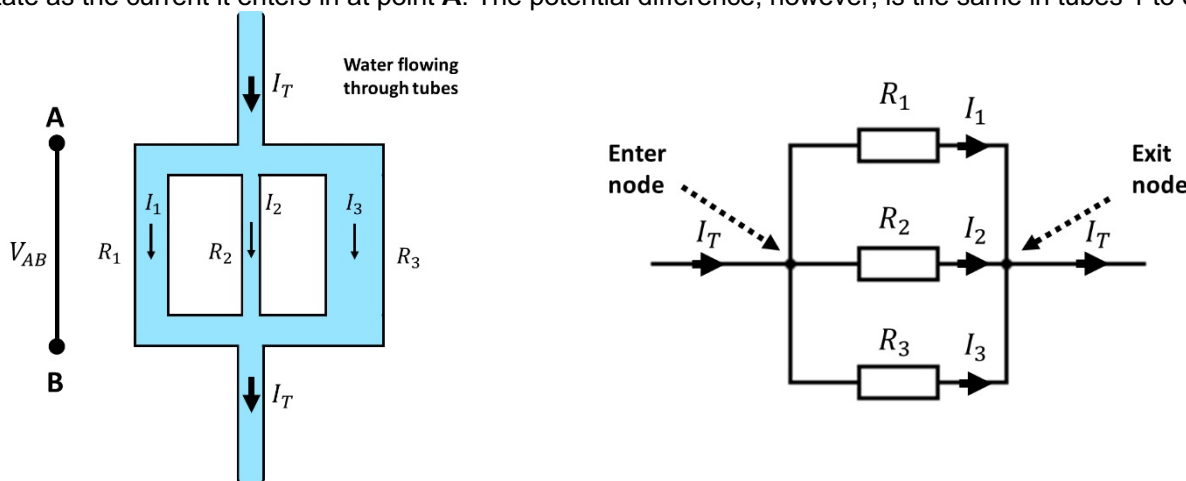
$$R_{\text{Series}} = R_T = R_1 + R_2 + \dots + R_n$$

The total resistance is also called the *equivalent resistance*, and we can reduce the multi-resistor circuit to one equivalent resistor.



CONTENT – PARALLEL CIRCUITS

For resistors connected in parallel the voltage flowing through each resistor is the same but the current is divided, which is the opposite behaviour to resistors connected in series. Consider the image of water tubes below where the flow of water splits into three tubes of different widths (point **A**). The amount of water flowing through tube 1 to 3 depends on the width of the tubes. As the water reaches the exit junction, point **B**, the current flow returns to the same state as the current it enters in at point **A**. The potential difference, however, is the same in tubes 1 to 3.



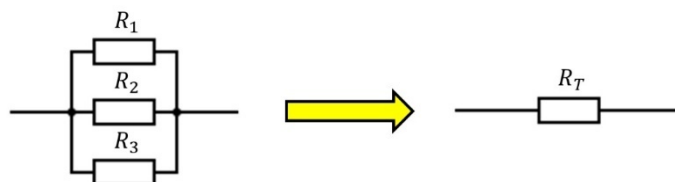
Similarly, for an electric circuit, the sum of the electric current flowing through each resistor connected in parallel is equal to the total current flowing into or out of the node/junction. This behaviour is known as the **Kirchoff's Current Law (KCL)**, which states that *the total current entering a node is equal to the total current exiting the node* (i.e. conservation of charge). Mathematically this is written as

$$I_{\text{Parallel}} = I_T = I_1 + I_2 + \dots + I_n$$

The equivalent resistance of resistors connected in parallel is calculated by summing up the reciprocal of each resistance value

$$\frac{1}{R_{\text{Parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

Transforming the resistors connected in parallel into one equivalent resistor



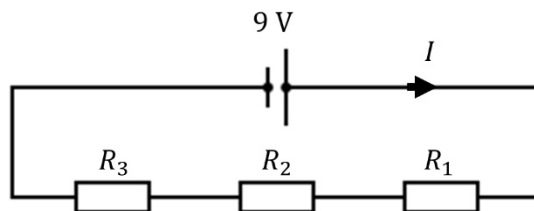
SUMMARY

The table below summarises the general behaviour of current and voltage in circuits.

Resistor Connection	Voltage	Current
Series	Divided	Constant
Parallel	Constant	Divided

WORKED EXAMPLE – SERIES CIRCUIT

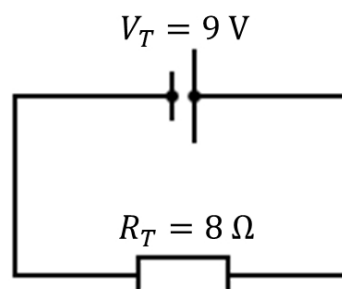
Find the voltage running through each resistor for the series circuit on the right. The resistance values are $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$ and $R_3 = 5\ \Omega$.



⇒ We have three resistors connected in series, and according to *Kirchoff's Current Law*, the current passing through each resistor is the same. Thus, if we know the total current is running through the circuit, we know the current running through each resistor.

⇒ We start by combining the three resistors connected in series into one equivalent resistor

$$\begin{aligned} R_T &= R_1 + R_2 + R_3 \\ &= 1\ \Omega + 2\ \Omega + 5\ \Omega \\ &= 8\ \Omega \end{aligned}$$



⇒ The total current running through the whole circuit is:

$$\begin{aligned} V &= IR \\ I_T &= \frac{V_T}{R_T} = \frac{9\ \text{V}}{8\ \Omega} = 1.125\ \text{A} \end{aligned}$$

⇒ Thus, the voltage across each resistor is:

$$\begin{aligned} V_{R_1} &= I_T R_1 \\ &= 1.13\ \text{A} \times 1\ \Omega \\ &= 1.125\ \text{V} \end{aligned}$$

$$\begin{aligned} V_{R_2} &= I_T R_2 \\ &= 1.13\ \text{A} \times 2\ \Omega \\ &= 2.250\ \text{V} \end{aligned}$$

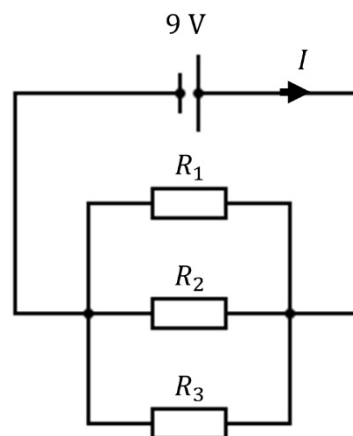
$$\begin{aligned} V_{R_1} &= I_T R_1 \\ &= 1.13\ \text{A} \times 5\ \Omega \\ &= 5.625\ \text{V} \end{aligned}$$

⇒ To check if we calculated the voltage correctly, make sure the sum of the voltages across each resistor is equal to 9.0 V (*Kirchoff's Voltage Law*).

$$\begin{aligned} V_T &= V_{R_1} + V_{R_2} + V_{R_3} \\ &= 1.125\ \text{V} + 2.250\ \text{V} + 5.625\ \text{V} \\ &= 9.0\ \text{V} \end{aligned}$$

WORKED EXAMPLE – PARALLEL CIRCUIT

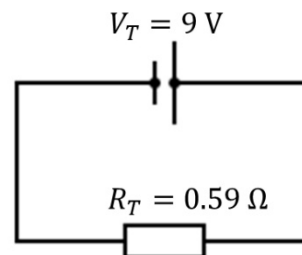
Find the current running through each resistor for the parallel circuit on the right. The resistance values are $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$ and $R_3 = 5\ \Omega$.



⇒ The three resistors are connected in parallel and the current entering and exiting the node of the resistors is the same. The voltage across each resistor is the same.

⇒ We combine the three resistors into one equivalent resistor

$$\begin{aligned}\frac{1}{R_T} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \\ &= \frac{1}{1\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{5\ \Omega} \\ &= \frac{17}{10\ \Omega}\end{aligned}\quad R_T = \frac{1}{17/10\ \Omega} = 0.59\ \Omega$$



⇒ The total current running through the whole circuit is:

$$V = IR$$

$$I_T = \frac{V_T}{R_T} = \frac{9\ \text{V}}{0.59\ \Omega} = 15.3\ \text{A}$$

⇒ The voltage running through each resistor is 9 V, and thus the current through each resistor is:

$$\begin{aligned}I_{R_1} &= \frac{V_T}{R_1} \\ &= \frac{9\ \text{V}}{1\ \Omega} \\ &= 9.0\ \text{A}\end{aligned}$$

$$\begin{aligned}I_{R_2} &= \frac{V_T}{R_2} \\ &= \frac{9\ \text{V}}{2\ \Omega} \\ &= 4.5\ \text{A}\end{aligned}$$

$$\begin{aligned}I_{R_3} &= \frac{V_T}{R_3} \\ &= \frac{9\ \text{V}}{5\ \Omega} \\ &= 1.8\ \text{A}\end{aligned}$$

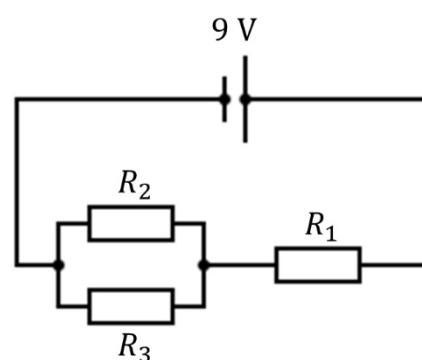
⇒ Check that the sum of the currents is equal to the total current:

⇒ Observe that adding resistances (e.g. 1 Ω, 2 Ω and 5 Ω) in parallel decreases the total resistance (0.59 Ω)

$$\begin{aligned}I_T &= I_1 + I_2 + I_3 \\ &= 9.0\ \text{A} + 4.5\ \text{A} + 1.8\ \text{A} \\ &= 15.3\ \text{A}\end{aligned}$$

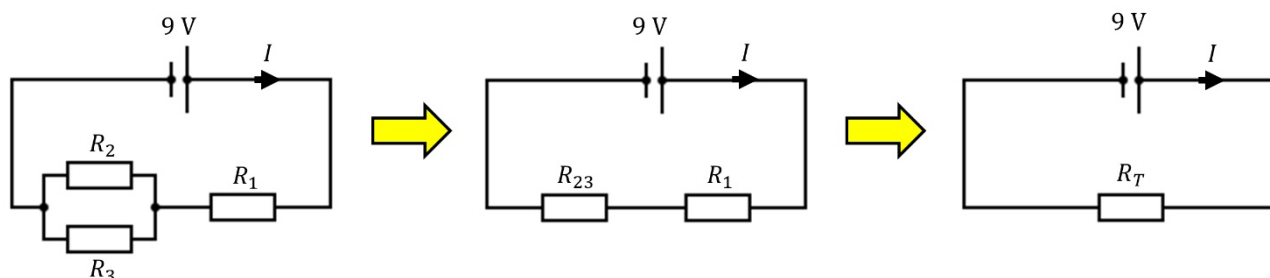
WORKED EXAMPLE – COMBINED SERIES AND PARALLEL CIRCUIT

Find the voltage and current running through each resistor for the series-parallel circuit on the right. The resistance values are $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$ and $R_3 = 5\ \Omega$.



⇒ For this circuit, the current flows through R_1 in series and then splits on a node to R_2 and R_3 in parallel. Much like solving the series and parallel only circuit we want to reduce the circuit into a one resistor circuit.

⇒ We first combine the resistors connected in parallel and then combine the resistors in series. Pictorially we perform the transformation below.





⇒ Below is the working out of combining the resistors as shown above

$$\begin{aligned}\frac{1}{R_{23}} &= \frac{1}{R_2} + \frac{1}{R_3} \\ &= \frac{1}{2\ \Omega} + \frac{1}{5\ \Omega} \\ &= \frac{7}{10\ \Omega}\end{aligned}$$

$$\begin{aligned}R_{23} &= \frac{1}{\frac{7}{10\ \Omega}} \\ &= 1.43\ \Omega\end{aligned}$$

$$\begin{aligned}R_T &= R_{23} + R_1 \\ &= 1.43\ \Omega + 1\ \Omega \\ &= 2.43\ \Omega\end{aligned}$$

⇒ The total current is:

$$\begin{aligned}V &= IR \\ I_T &= \frac{V_T}{R_T} = \frac{9.0\ \text{V}}{2.43\ \Omega} = 3.70\ \text{A}\end{aligned}$$

⇒ Once we have the total current, we work our way back and expand the circuit to get the current and voltages

⇒ The current is constant for resistors connected in series. Thus 3.70 A flows through both R_{23} and R_1 .

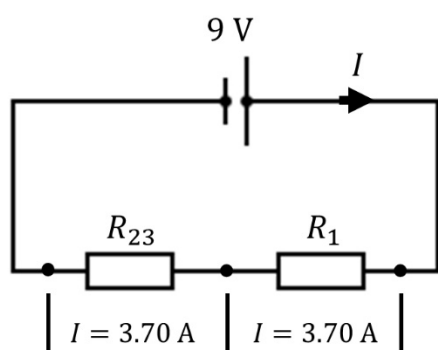
⇒ The voltage across R_{23} and R_1 is:

$$\begin{aligned}V_{R_{23}} &= I_T R_{23} \\ &= 3.70\ \text{A} \times 1.43\ \Omega \\ &= 5.30\ \text{V}\end{aligned}$$

$$\begin{aligned}V_{R_1} &= I_T R_1 \\ &= 3.70\ \text{A} \times 1\ \Omega \\ &= 3.70\ \text{V}\end{aligned}$$

⇒ Check sum of voltage is equal to 9.0 V:

$$\begin{aligned}V_T &= V_{R_{23}} + V_{R_1} \\ &= 5.30\ \text{V} + 3.70\ \text{V} \\ &= 9.0\ \text{V}\end{aligned}$$

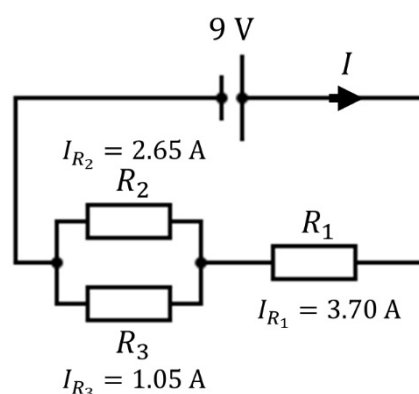


⇒ Now that we have the voltage running through R_{23} we can calculate the current through R_2 and R_3 .

$$\begin{aligned}I_{R_2} &= \frac{V_{R_{23}}}{R_2} = \frac{5.29\ \text{V}}{2\ \Omega} = 2.65\ \text{A} \\ I_{R_3} &= \frac{V_{R_{23}}}{R_3} = \frac{5.29\ \text{V}}{5\ \Omega} = 1.05\ \text{A}\end{aligned}$$

⇒ Check that the sum of the currents is equal to 3.70 A

$$\begin{aligned}I_{R_{23}} &= I_{R_2} + I_{R_3} \\ &= 2.65\ \text{A} + 1.05\ \text{A} \\ &= 3.70\ \text{A}\end{aligned}$$



⇒ Summary of the current and voltages:

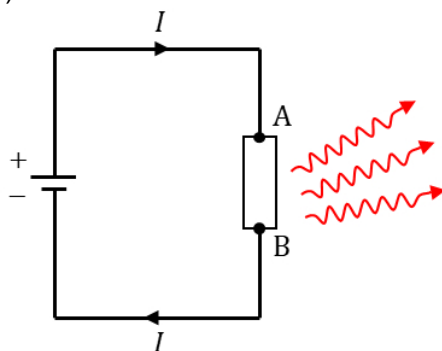
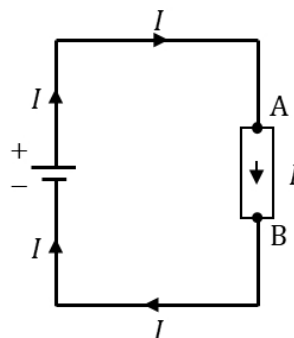
Voltage (V)		Current (A)	
V_1	3.70	I_1	3.70
V_2	5.30	I_2	2.65
V_3	5.30	I_3	1.05
V_T	9.00	I_T	3.70



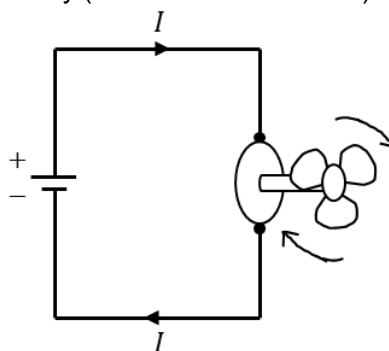
HEATING EFFECTS OF ELECTRIC CURRENTS

CONTENT – CONSERVATION OF ENERGY

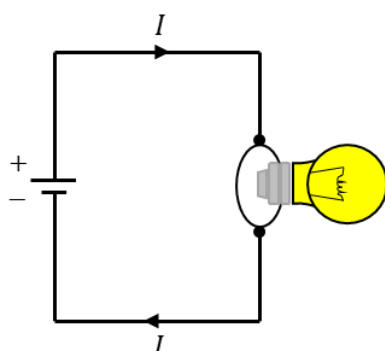
An electric current that passes through an electric component will have its potential energy dropped by a certain amount (potential difference or voltage). Following the circuit on the right, the electric current flows from a source starting from the positive terminal. As the current passes through the resistor, the potential energy of the electric current drops between point A and B. Finally, the current flows to the negative terminal completing the circuit. Following the law of conservation of energy, the drop in potential energy from point A to B must be transformed into a different kind of energy (the energy loss across the resistor cannot be destroyed). For a resistor, the electrical energy is transferred into heat (**electrical to thermal**). Other devices connected similarly will convert the electrical energy differently. Examples include electric current passing through a motor connected to a mechanical load, light bulb and a storage battery. The electrical energy is transferred as work done on the mechanical load (**electrical to mechanical**), converted to light (**electrical to electromagnetic**) and transferred to a stored chemical in the battery (**electrical to chemical**).



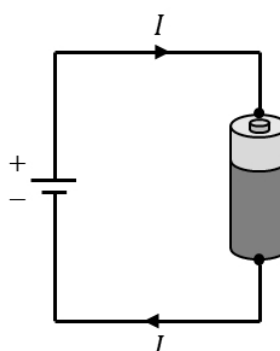
Electrical → Heat



Electrical → Mechanical



Electrical → Light



Electrical → Chemical

CONTENT – POWER IN CIRCUITS

The transfer of energy is quantitatively described with a variable called power, P . The units for P is joule/sec or Watts, hence it is a measure of the amount of energy transferred per second or unit time. The power transferred from electric current is given by the voltage multiplied by the current.

$$P = VI \quad (1)$$

To check if the expression above is correct, we will look at the units of both voltage and current. Voltage is defined as the amount of work done per unit charge (joule/coulomb), and current is the flow of charge per unit time (coulomb/sec). Thus, if we have 1 Volt and 1 Amp the multiplication of the two is

$$1 \frac{\text{Joule}}{\text{Coulomb}} \times 1 \frac{\text{Coulomb}}{\text{sec}} = 1 \frac{\text{Joule}}{\text{sec}} \text{ or } 1 \text{ Watt}$$

which is the unit for power. We can combine Ohm's law with the power equation above to get the dissipation rate of electrical energy across a resistor. Ohm's law is usually written as $V = IR$ and if we substitute Ohm's law into equation (1) we get

$$P = (IR) \times I = I^2 R \quad (2)$$

If instead, we write Ohm's law as $I = V/R$ then power dissipated across the resistor is

$$P = V \times \left(\frac{V}{R}\right) = \frac{V^2}{R} \quad (3)$$

In the case of a series circuit, where current is the same across all resistors, the resistor with the greater resistance will have a greater power dissipation. In other words, if current I stays the same in equation 2), then the greater the resistance, the greater the power dissipation. In the case of a parallel circuit, however, where voltage across the resistors stays the same, the resistor with the greater resistance will have a smaller power dissipation. This can be explained by visualising voltage V as a constant in equation 3) and varying the resistance R .

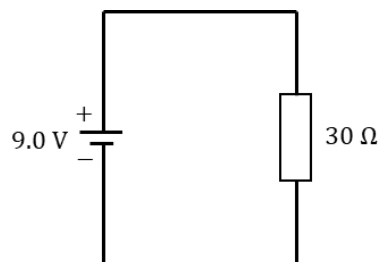
It is important to remember that equation (1) is the rate of transfer of electrical energy to all forms of energy. Equation (2) and (3) gives the rate of transfer of electrical energy to thermal energy for an electrical component with resistance. This behaviour of electric current in circuits is used in the design of electric heaters.

QUESTION 1

A $30 \, \Omega$ resistor is connected to a $9.0 \, \text{V}$ battery as shown on the circuit on the right.

- How much work is done on moving an electron across the resistor?
- What is the rate of energy transferred into heat across the resistor?
- How much thermal energy is produced in 1 hour?

ans: (a) $9.0 \, \text{J}$ (b) $2.7 \, \text{W}$ (c) $9.7 \, \text{kJ}$



QUESTION 2

Heat is generated in a resistor at a rate of $50 \, \text{W}$. If the current running through the resistor is $1.5 \, \text{A}$ what is the resistance value of the resistor? What is potential difference across the resistor?

ans: $R=22.2 \, \Omega$ and $V=33.3 \, \text{V}$

QUESTION 3

The standard voltage in Australia is $230 \, \text{V}$ and if a $1000 \, \text{W}$ heater is plugged into an outlet, find:

- The amount of current flowing through the heater
- The resistance of the heater
- The total cost of continuously running the bulb for seven days if it cost $12.5 \, \text{cents}/(\text{kW.h})$

ans: (a) $4.3 \, \text{A}$ (b) $53.5 \, \Omega$ (c) $\$21.00$

QUESTION 4 INTERNET RESEARCH

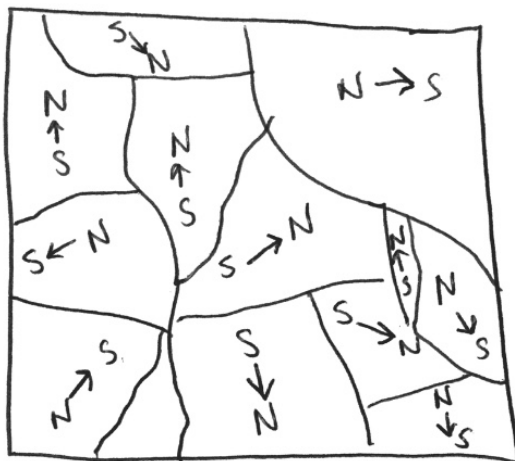
- Investigate the physics of fireworks in terms of energy conversions. How many energy conversions can you list?
- Research the power consumption of common household appliances. Pick one appliance and study the energy rating of that appliance. What does the star rating mean?



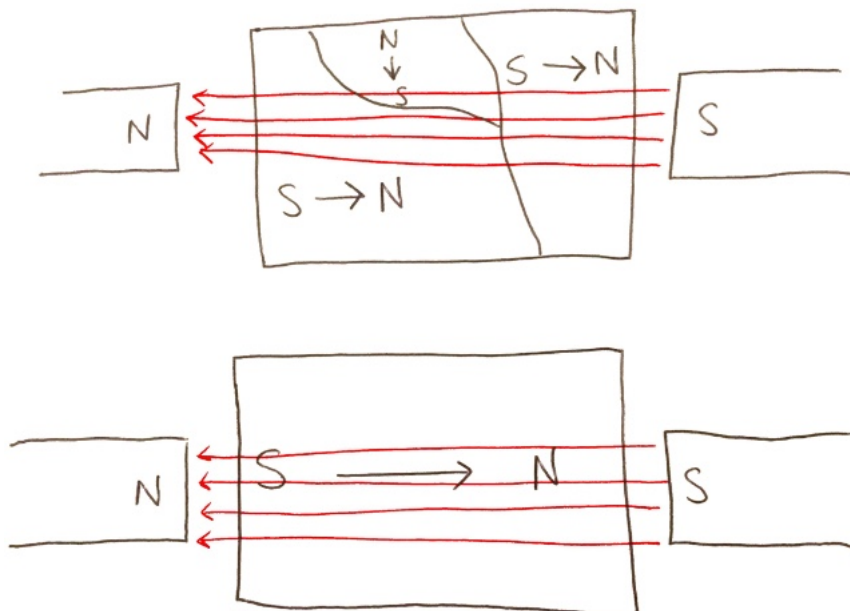
FERROMAGNETS

CONTENT

Ferromagnets are materials that have special magnetic properties. Iron is the most common ferromagnetic material, but other metals, including rare earths, can be used. By themselves, ferromagnetic materials are not magnets and on a large scale they show no overall magnetisation. But on a smaller scale, if a magnet is placed near a ferromagnetic material, it can make regions of aligned magnetisation. These are called domains. Each domain acts as a small internal bar magnet and the alignment of each domain is random compared to the other domains, as shown below. As a result on a large scale the internal magnetic fields mostly cancel each other out and the overall magnetisation is minimal.



Ferromagnetic material can be made into a permanent bar magnet by placing it within a strong magnetic field and heating it. This causes all the magnetic fields within the domains to align and produce an overall bar magnet, as in the picture below. Note the north and south poles of the ferromagnet are opposite to the north and south poles of the original magnets. That is why iron is attracted to magnets. When the domains align in a magnetic field, they align in such a way that the ferromagnetic material is attracted to the magnets.



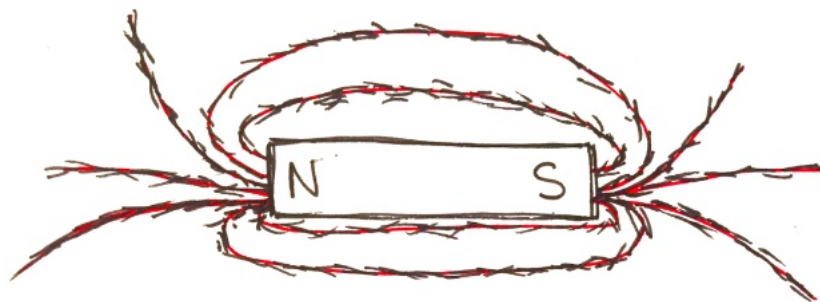
The domains act like mini internal bar magnets because of the coupling of electron spins in neighbouring atoms. The electrons in every atom have magnetic moments. In each domain the magnetic moments of the electrons in all the atoms tend to point in the same direction, giving the domain an overall magnetic moment in that direction. So when the ferromagnetic material is exposed to a strong magnetic field, all the magnetic moments in each domain align.

EXAMPLE

Using your understanding of ferromagnetic material, explain how iron filings (small particles of iron) can be used to display magnetic fields.

⇒ So, firstly, we know that iron is a ferromagnetic material so it's very responsive to magnets. They also have domains which have aligned magnetisation. When a magnet is placed inside or near the iron filings, the domains within the iron filings become aligned. Due to the shape of the iron filings, their lowest energy configuration has the magnetisation aligned along the longest side of each individual iron filing. Like all ferromagnets in the presence of a strong magnetic field, the ferromagnets align in the opposite direction of the magnetic field. This means they wish to stay within the magnetic field. The iron filings are also small enough that the friction of the surface is usually too small to stop the iron filings from moving. So each individual iron filing will move so that its longest side is aligned in the direction of the magnetic field.

Thus, the iron filings move to point along the direction of the magnetic field, allowing us to see what the magnetic field lines look like. In the diagram below, the red lines are the magnetic field lines and the small black lines represent the iron filings.



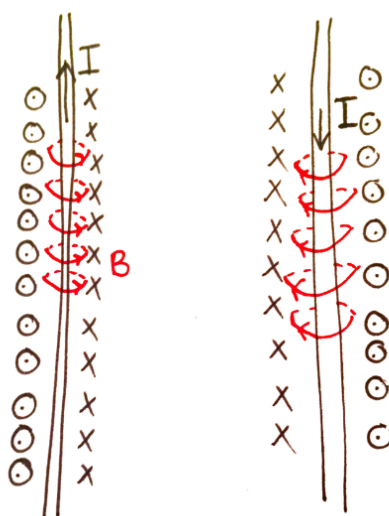


MAGNETIC FIELDS OF WIRES AND SOLENOIDS

CONTENT

A current-carrying wire produces a magnetic field. We can use this property to generate strong magnetic fields and convert electrical energy into mechanical energy. In this worksheet, we will first look at the magnetic fields of a current-carrying wire and then of a solenoid.

The direction of the induced magnetic field, B , from a current-carrying wire depends on the direction of the current, I , itself. We use the Right Hand Palm Rule to figure out the direction. By pointing your thumb in the direction of the current and curling your fingers around the wire, the direction of the curl represents the direction of the magnetic field. To draw the magnetic field, we use 'X's to represent the magnetic field going into the page and circles with dots to represent the field coming out of the page. An easy way to remember this is that the 'X's look like the feathers on the back of an arrow that is pointing away from you, so the direction is away from you.



In the example on the left, the magnetic field is going into the page on the right and out of the page on the left. When the direction of the current is flipped so is the direction of the magnetic field. To calculate the strength of the magnetic field we use the equation

$$B = \frac{\mu_0 I}{2\pi r},$$

where B is the magnetic field strength, μ_0 is the permeability of free space ($4\pi \times 10^{-7}$), I is the current and r is the minimum distance to the wire. Thus the stronger the current and the smaller the radius of the wire, the stronger the magnetic field.

If we tightly wrap a long wire around a hollow cylinder we create a solenoid. The coil must be much longer than the diameter. A magnetic field is produced when the current passes through the coil of wire. Since the wires are tightly wound, the magnetic field inside the coil is essentially uniform. It also acts like a bar magnet since the magnetic field outside the coil is weak. As in a single wire, the direction of the magnetic field depends on the direction of the current in the wire.

In the solenoid we can use the Right Hand Rule (slightly different to the Right Hand Palm Rule) to determine the direction of the magnetic field. This time, our fingers wrap around the coil in the direction of the current flow and our thumb points in the direction of the magnetic field. To calculate the strength of the magnetic field for a solenoid we use the equation

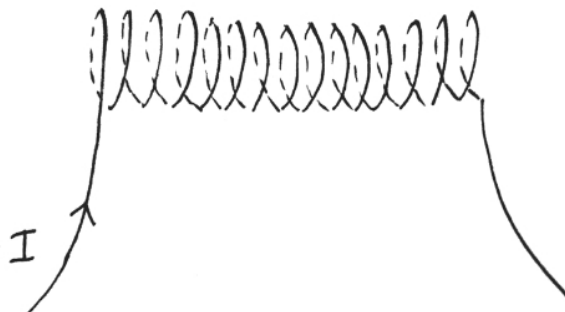
$$B = \frac{\mu_0 NI}{L},$$

where N is the number of coils, I is the current and L is the length of the solenoid.

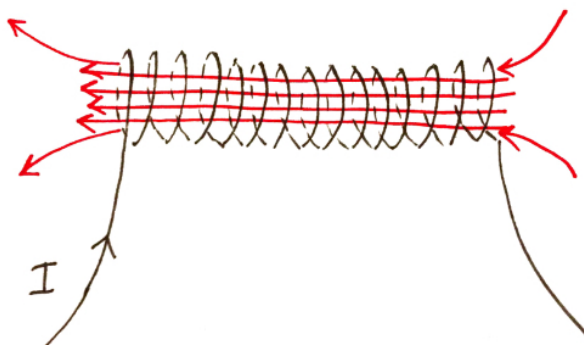


EXAMPLE

A wire is wound around a hollow cylinder 500 times to create a solenoid. The solenoid has a length of 0.5m. A current of 12A is passing through the wire. In the diagram below, add the direction of the magnetic field. What is the magnitude of the magnetic field?



- ⇒ Firstly, adding in the direction of the magnetic field. We use the Right Hand Rule, wrapping our fingers around the coil in the direction of the current, then our thumb is pointing in the direction of the magnetic field. Thus we find:



- ⇒ Now, to calculate the magnitude of the magnetic field. Let's start by writing down all the variables we have so we can sub them into the equation:

Variable	Value
μ_0	$4\pi \times 10^{-7}$
I	12A
L	0.5m
N	500 turns
B	?

$$\begin{aligned} B &= \frac{\mu_0 N I}{L} \\ &= \frac{4\pi \times 10^{-7} \times 500 \times 12}{0.5} \\ &= 0.01508 \dots \\ &= 0.02 \text{ T} \end{aligned}$$