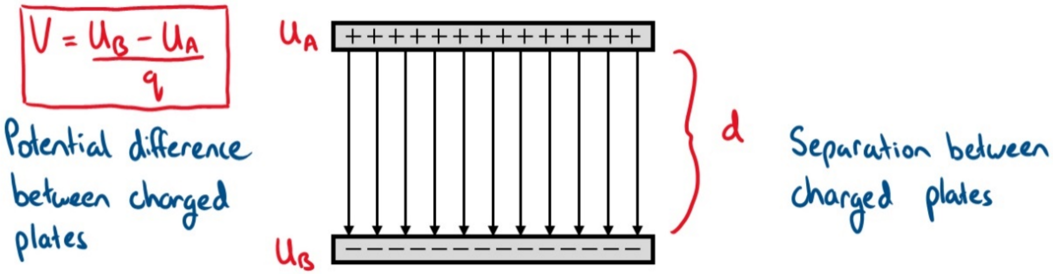
CHARGED PARTICLES IN AN ELECTRIC FIELD 1

CONTENT – CHARGED PARTICLES

In *module 4* we have looked at the definition of the voltage. Voltage is the difference in potential energy between two points. For parallel charged plates the potential difference is illustrated below.

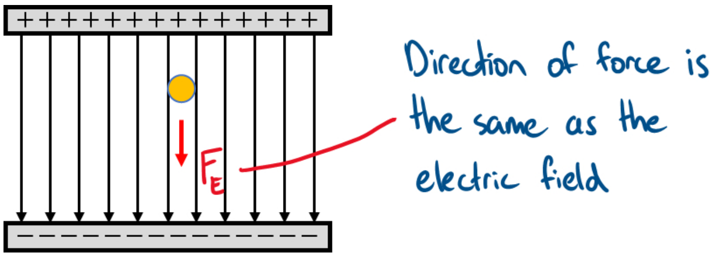


The electric field between the parallel plates is uniform and is defined as the potential difference divided by the distance between the plates.

$$|E| = -\frac{V}{d} \quad (1)$$

The units for the electric field defined above is Volts per metre (V/m).

Let's now suppose a positively charged particle is placed in between the parallel plates. The particle will experience a force resulting from the electric field, and the direction will be parallel to the field.



If the charge on the particle is q the force on the particle from the electric field is

$$F_{charge} = qE \quad (2)$$

If we equate this force to the force of a moving object (i.e. $F = ma$) we, can determine the acceleration of the charged particle when it is inside the electric field.

$$F_{net} = F_{charge}$$
$$ma = qE$$
$$a = \frac{qE}{m} \quad (3)$$

The acceleration above is the acceleration of the particle as it moves towards the negative plate and is analogous to the acceleration due to gravity (i.e. $g = 9.8\ m/s^2$). Meaning, the charged particle is moving like an object falling in a gravitational field. If the charge is negative then ,the direction will be upwardly instead.

In addition to the force, we can calculate the work done on the particle from the potential difference between the two plates

$$W = qV \quad (4)$$

We can also define the work in terms of the electric field using equation (1) and arrive with the expression below

$$W = qEd \quad (5)$$

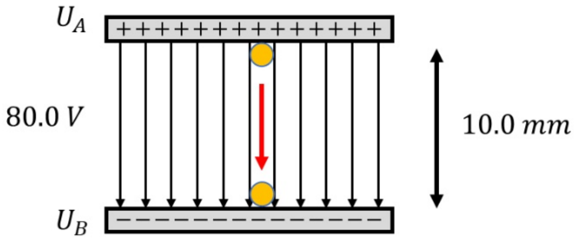
Now that we have defined the expression for work we can obtain the velocity of the particle. Recall the *work-energy theorem* that states work done on a particle is equal to the change in kinetic energy (i.e. $W = 1/2\, mv^2$). With this definition the velocity of the charged particle is

$$W = KE$$
$$qEd = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{2qEd}{m}} \quad (6)$$

Note: the terms Ed can be replaced with V in the velocity equation above.

Worked Example

A positively and negatively charged plates are separated by 10.0 mm as shown on the right. If the potential difference is 80.0 V what is the velocity of a proton as it hist the negative plate? What is the velocity if the potential difference is 1.0 V instead? (mass of proton = 1.67 x 10^{-27} kg and charge = 1.6 x 10^{-19} C).



⇒ To determine the velocity, we start with the work-energy theorem to arrive at an expression for velocity

$$W = KE$$
$$qEd = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{2qEd}{m}}$$

⇒ The electric field can be written in terms of the potential difference thus,

$$V = Ed \rightarrow v = \sqrt{\frac{2qV}{m}}$$

⇒ We now have an expression for velocity where we have all the numbers.

$$v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} C \times 80.0\, V}{1.67 \times 10^{-27} kg}}$$
$$= 1.24 \times 10^5\ m/s$$

⇒ The velocity above is extremely fast and to put it in perspective it is 375x the speed of sound.

⇒ If the potential difference is 1.0 V, then the velocity is

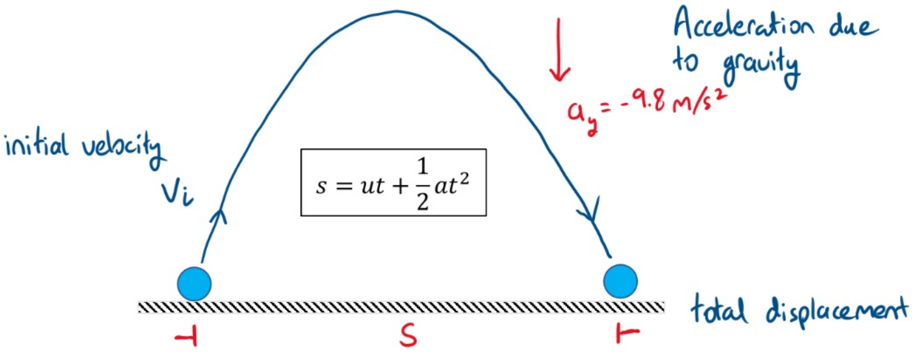
$$v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} C \times 1.0\, V}{1.67 \times 10^{-27} kg}}$$
$$= 1.38 \times 10^4\ m/s$$

⇒ So even with only 1.0 V, the velocity is still large because the mass of the proton is very small.

CHARGED PARTICLES IN AN ELECTRIC FIELD 2

CONTENT – MOVING CHARGED PARTICLES

Previously we looked at what happens to a charged particle when it is placed in a uniform electric field. However, the question that comes up after is what happens to the charged particle if it moves across the parallel plates. The acceleration due to the electric field is constant and depends on the electric field and the properties of the charged particle. Since it is constant, we can use 2D kinematics that we learned in *module 1* (i.e. projectile motion). Suppose we have an electron placed just outside the parallel plates, the position in two dimensions is (x,y). The initial velocity of the electron is v_i. Recall that the displacement of a moving object for a constant acceleration and given the initial velocity is



If we decompose the displacement in two dimensions we get

$$x = v_x t + \frac{1}{2}a_x t^2 = v_x t$$
$$y = v_y t + \frac{1}{2}a_y t^2 = \frac{1}{2}a_y t^2$$

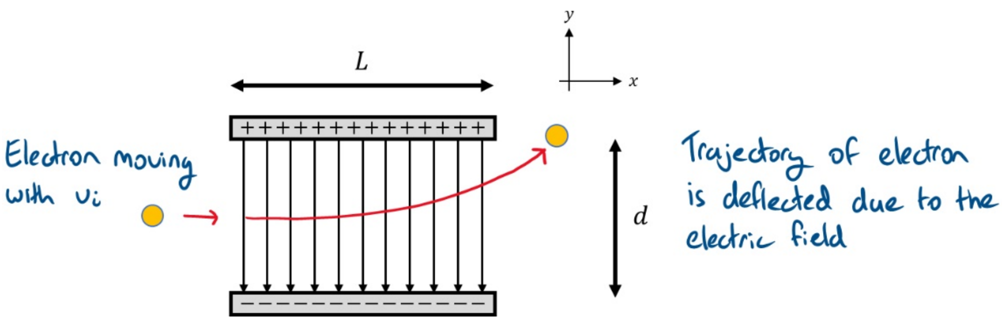
We arrive at the expression above by taking acceleration in the x-direction as zero ($a_x = 0$) and the velocity in the y direction also zero ($v_y = 0$). For the electron moving through the parallel plates, the initial velocity is $v_x = v_i$ and the acceleration from the electric field is $a_y = qE/m$. Thus the position of the electron moving through the parallel plates at a given time is

$$x = v_i t$$
$$y = \frac{qE}{2m}t^2$$

If we rearrange the equation for x to make t the subject ($t = x/v_i$) we can substitute this term to the equation for y. The displacement of the electron in the y direction is then

$$y = \frac{qE}{2m}\left(\frac{x}{v_i}\right)^2 = \left(\frac{qE}{2mv_i^2}\right)x^2$$

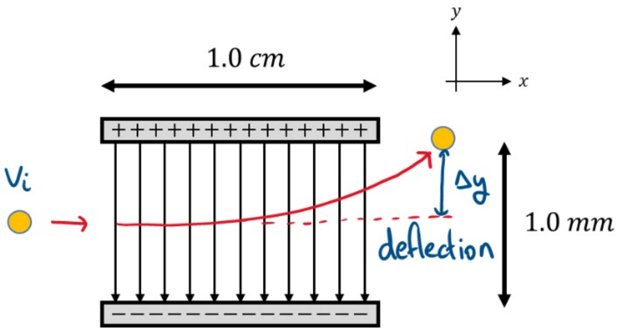
The terms in the parenthesis are constant, and the trajectory of the electron is parabolic. The sharpness of the parabola depends on the electric field and the mass and velocity of the particle. An illustration of the electron's trajectory is shown below.



The electron is deflected in the y-direction because of the electric field, and it deflects towards the positively charged plate because of the negative charge on the electron. If the particle is a proton instead, then the particle will be deflected towards the negative plate.

WORKED EXAMPLE

An electron moves through a uniform electric field produced by two parallel conducting plates separated by 1.0 mm and 1.0 cm in length shown in the diagram to the right (not to scale). The initial velocity of the electron is 1.5 x 10^6 m/s and the potential difference between the plates is 9.0 V. Find (a) the acceleration due to the electric field, (b) the time taken for the electron to pass through the field and (c) the deflection of the electron in the y-direction.



a)

⇒ We first need to determine the electric field produced by the conducting plates

$$E = \frac{V}{d} = \frac{1.0\ V}{1.0 \times 10^{-3}\ m} = 1000\ V/m$$

⇒ Given the electric field, we can calculate the acceleration of the particle in the y-direction

$$a_y = \frac{qE}{m} = \frac{1.6 \times 10^{-19}\ C \times 1000\ V/m}{9.11 \times 10^{-31}\ kg}$$
$$= 1.76 \times 10^{14}\ m/s^2$$

b)

⇒ The total displacement in the x-direction is 1.0 cm, then the total time for the electron to move through the plates is is

$$x = \frac{v_i}{t} \rightarrow t = \frac{x}{v_i}$$
$$t = \frac{1.0 \times 10^{-2}\ m}{1.5 \times 10^{6}\ m/s} = 6.67 \times 10^{-9}\ s$$

c)

⇒ For the deflection in y, we use the kinematic displacement equation

$$y = \frac{1}{2} a_y t^2$$
$$= \frac{1}{2} \times 1.76 \times 10^{14}\ m/s^2 \times (6.67 \times 10^{-9}\ s)^2$$
$$= 3.9 \times 10^{-3}\ m$$

⇒ Therefore, the electron is deflected in the y-direction by 3.9 mm as a result of the electric field

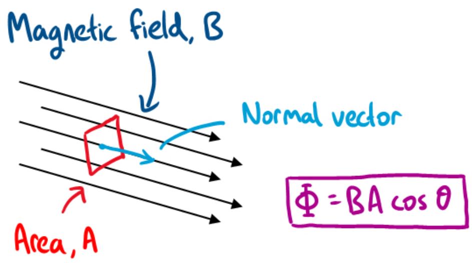
QUESTION – INTERNET RESEARCH

Explain how the phenomenon of charged particles moving in an electric field is used in cathode ray tubes (CRT) and subsequently how is it exploited in old CRT televisions.

ELECTROMAGNETIC INDUCTION

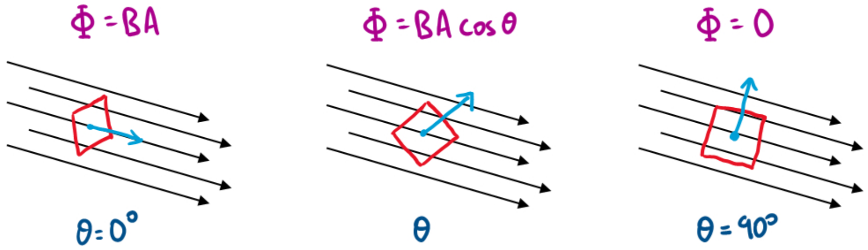
CONTENT – MAGNETIC FLUX

Magnetic fields, written with the symbol B, are measured in units of Telsa (T). To describe magnetic fields, we introduce a quantity called the magnetic flux. The magnetic flux quantifies the total magnetic field passing through a certain surface area. Following the diagram on the right, the total magnetic field coming through the small rectangular area is



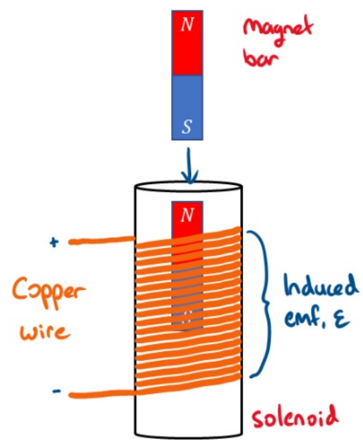
$$\Phi = B_{\parallel}A = BA\cos\theta \qquad (1)$$

where Φ is the magnetic flux having units of Weber (Wb). The angle in the cosine is the angle between the normal vector of the area and the magnetic field line. If the area is perpendicular to the direction of the magnetic field, $\theta = 0°$, then the cosine term becomes one, which means that the magnetic flux is at a maximum. When the area is tilted by an angle other than zero then the magnetic flux through the area will be less than the maximum. As a final case, if the angle is 90º, then the magnetic flux through the area is zero.



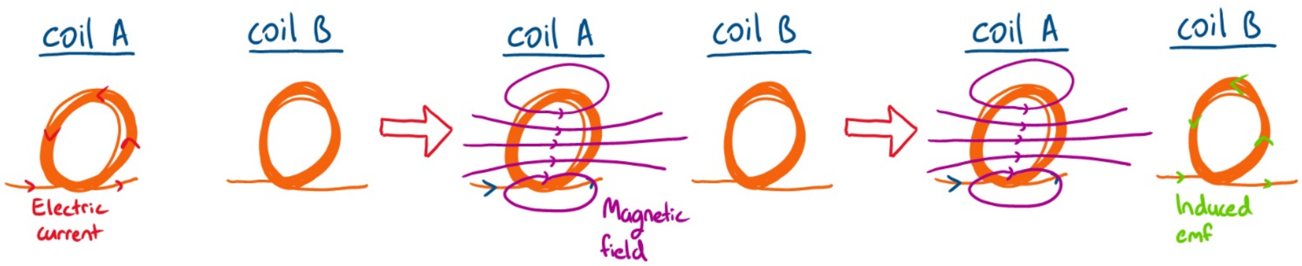
CONTENT – ELECTROMAGNETIC INDUCTION

Suppose we have a solenoid with a copper wire wrapped around N times. If we move a magnetic bar through the solenoid, the magnetic flux will change along the way. This change in magnetic flux induces an electromotive force (emf) ε to the solenoid. The electromotive force is the potential difference and measured in units of Voltage (V). This induction of emf to the solenoid is known as Faraday's law of induction. The direction of the emf will be opposite to the magnetic flux and is known as Lenz's law. Combining Faraday's and Lenz's laws the induced emf due to changing magnetic flux is



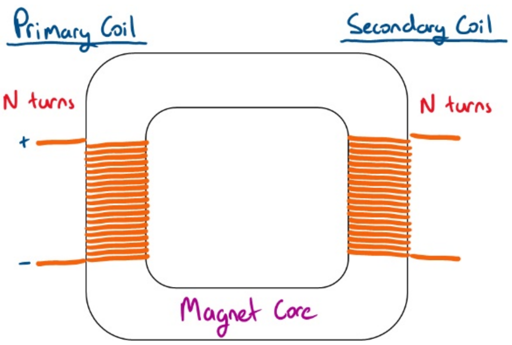
$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} \qquad (2)$$

Suppose now we have two copper coils positioned side-by-side as shown below (basically two solenoids). When a current passes through coil one, a magnetic field is produced around the coil. If coil B is close to coil A, then the magnetic field produced by coil one will move the electrons in coil B. Following the induction laws, coil two will produce an emf due to the change in magnetic flux produced by coil A. Thus, energy is transferred from one coil to another propagated through space.



CONTENT – TRANSFORMERS

The induction laws are used in devices such as transformers. Transformers transfer electricity between two or more circuits using electromagnetic induction (i.e. not connected by wires). Consider the magnet core below with copper wires wrapped around on each side with *N* turns.



The induced emf on the secondary coil depends on the ratio of the number of turns of the primary, N_p, to the secondary, N_s, coil. If the emf in the primary and secondary is V_p and V_s respectively, the relationship between two coils is written as

$$\frac{V_p}{V_s} = \frac{N_p}{N_s} \tag{3}$$

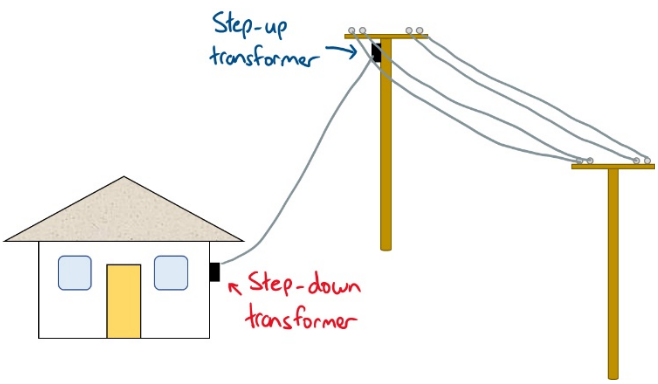
If the number of turns in the primary and secondary coils are the same, then the emf induced in the secondary coil is the same. This holds true for an ideal transformer where there is no loss of current due to resistance in the copper and no magnetic flux leakage. Following the law of conservation of energy, the power in the primary coil must be equal to the power out of the secondary coil (power going in must be equal to power going out). Since power from electric circuit can be calculated by multiplying voltage by the current ($P = VI$), the power through the coils is

$$V_p I_p = V_s I_s \tag{4}$$

Thus, if the emf in the secondary is smaller than the primary, then the current in the secondary is greater than the primary and vice-versa.

We have defined ideal transformers where there is no energy loss in transmission. However, real transformers are far from ideal, and the efficiency of the transmission is less than 100%. One reason for a loss of energy during transmission is an *incomplete flux linkage*. This happens when the magnetic flux produced by the first coil is not fully linked to the second coil reducing the emf induced. Another issue is the generation of *resistive heat* from *eddy currents*. Eddy currents can be reduced by using a better material or laminated core.

REAL WORLD APPLICATION – POWER LINES



Transformers are used in the distribution of electricity through high-voltage power lines. The strength of electric current drops over long distance and the signal need to be boosted again for the signal to continue. From equation (3), if the number of turns in the primary coil is smaller than the secondary coil then induced emf will be greater than emf in the primary. This is an example of a *step-up* transformer where the voltage is boosted. If the number of turns in the primary is greater than the secondary then the opposite will occur, i.e. reduces the induced emf. This is an example of a *step-down* transformer. Step-up transformers are used to boost signals for long-range power. In NSW the voltage in power lines can be as high as 66,000 V. The power outlet in houses output 240 V. Thus a step-down transformer is required to reduce the voltage down to the appropriate value.