MAXWELL AND CLASSICAL THEORY

CONTENT

In the 1800s, scientists were fascinated with two seemingly separate fields: electricity and magnetism. Preliminary research had been done relating magnetism to light, including the extraordinary work of Mary Somerville, who published a paper on the magnetisation of violet rays. But it was still largely not understood that these fields were related.

In 1865, Maxwell published his work combining electricity and magnetism in a new theory called electromagnetism. This work was summarised by four equations which combined and summarised all the current research of the time in both electricity and magnetism. Maxwell was able to mathematically connect magnetism and electricity by combining them in equations. For example, if we look at the following equation (don't worry about what the ∇ or $\times$ mean, we will go through each term separately):

$$\nabla \times \vec{B} = \mu_0(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t})$$

$$curl\ of\ magnetic\ field = static\ electric\ current + changing\ electric\ field$$

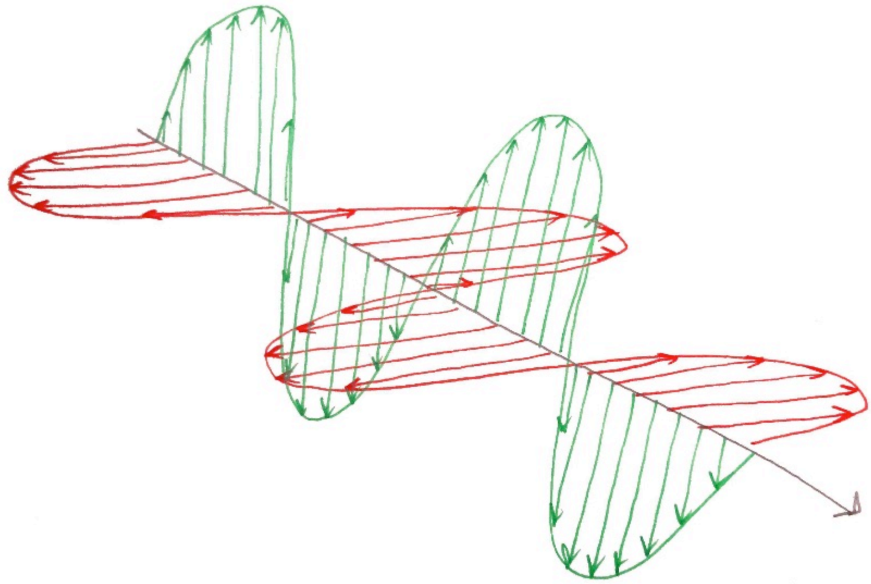
The term on the left, $\nabla \times \vec{B}$, refers to how the magnetic field curls around, much like when we use the right-hand rule to curl around a solenoid to determine the direction of the magnetic field. The first term on the right, $\mu_0\vec{J}$, is related to the electric current, while, the other term on the right-hand side, $\mu_0\epsilon_0 \frac{\partial \vec{E}}{\partial t}$, is related to how the electric field changes with time. Altogether, this equation means an electric current and/or a changing electric field produces a magnetic field.

While we don't need to know how to use this equation, because there are terms for both magnetic field and electric field, we can clearly see it relates the two. This equation summarises Maxwell's contribution to unifying electricity and magnetism.

From this equation, we can also predict the existence of electromagnetic waves. The two constants included are ϵ_0, the electric permittivity, and μ_0, magnetic permeability of free space. We have already seen these constants in Module 4: Electricity and Magnetism. They can be combined to determine the speed of light in a vacuum using the equation below:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

The combination of electricity and magnetism in this equation predicts not only that the speed of light is a constant, but that light is an electromagnetic wave. That is, it is an electric wave and a magnetic wave travelling together.



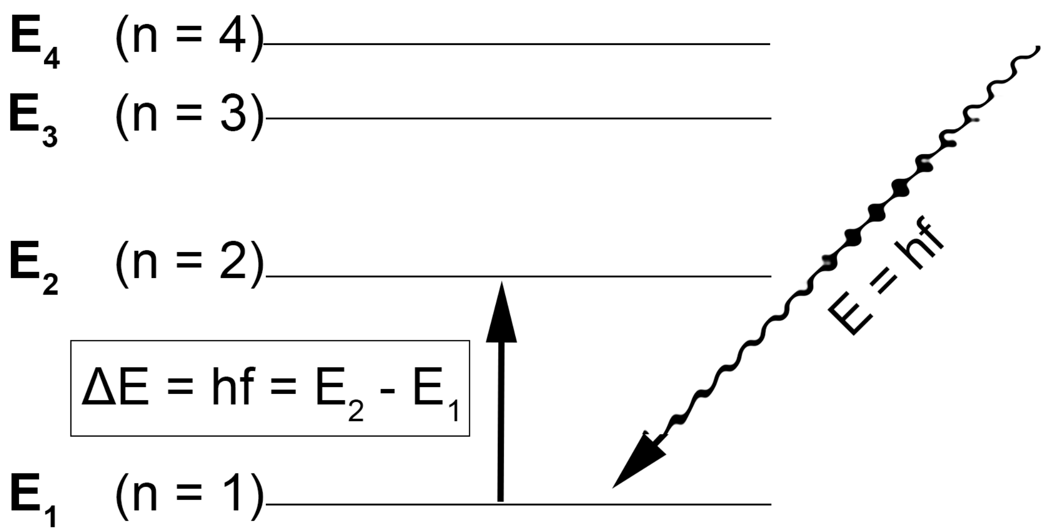
The prediction of electromagnetic waves comes directly from the definition of the speed of light in a vacuum. Thus, not only did Maxwell's equations predict light was part of the electromagnetic spectrum, they also predicted that the velocity of these waves, c, and set that velocity as a constant.

INVESTIGATION: SPECTROSCOPY

INTRODUCTION

The electrons orbiting around an atom have very specific energy levels they are allowed to be in. These electrons can absorb the energy of a photon to jump up an energy level, but this photon energy must be exactly the same as the energy difference between the two energy levels of the electron. Likewise, if an electron drops down an energy level it releases a photon which has the exact energy of the difference between the energy levels. As a result, we can see the bright lines called emission lines when the electron emits a photon and the absorption lines when the electron absorbs a photon.

The wavelengths of these lines depend on the element or molecule that has absorbed/emitted the photon and these lines are called the spectral lines or the spectra.



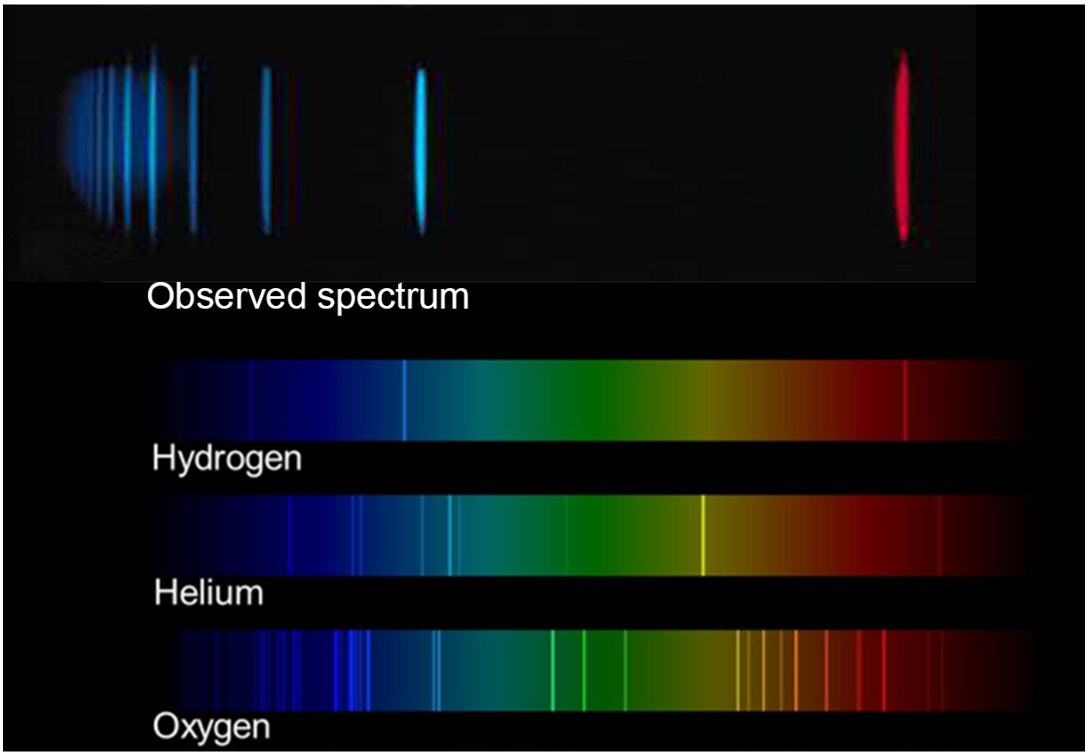
Absorption of a photon by an electron in an atom

In this experiment we will observe the spectra of several sources to see how they vary and what information we can glean from observing these features. We will also investigate the applications of spectral lines in both industry and astrophysics.

1. QUESTIONING AND PREDICTING

Let us think about the aim of this investigation:

1. What lines can we see when observing the different sources and how do these lines differ across the sources?
2. What could cause changes in the spectral lines?



How can we determine what element the observed spectrum matches?

Hypothesis

The wavelength where a spectral line is observed depends on the ______________________________

The multiple lines in a spectrum for one element are due to ____________________________________

2. Planning Investigation

This investigation has been planned for you. It is most suited to being performed by a whole class.

You will need at least one incandescent filament (a lamp filled with specific elements or molecules) and a spectrograph or spectroscope to spread the light into its component wavelengths.

1. Set up the incandescent filament so it is visible, and students can look at it with their spectroscopes.
2. Consider other sources of light to study with the spectroscope, these could include lights in the classroom or light from the sun.
3. Collect spectra of various elements to be able to compare and identify sources.

3. Conducting Investigation

For each element you observe, measure the wavelengths of the spectral lines that you see from your spectroscope. Record the wavelengths of all the spectral lines you observe.

Source	Measured Spectral Lines (wavelength nm)

Did you make any changes to the method? Did you have design problems to solve? Did you have some 'smart' ways of doing the investigation?

4. Processing and Analysis

For each of the sources you observed and your measured spectral lines, try and identify which element or elements are present in the source. To do this compare your measured wavelengths with the wavelengths of the spectral lines of known elements.

Are you able to identify all the sources?

What did you notice about the spectral lines of different sources? Where they the same? Similar?

If they were different, what could cause the differences?

Were there any difficulties identifying the sources? What were they? How could you fix them?

5. Problem Solving

All elements and molecules have very a specific set of spectral lines related to their electron orbits. Since these orbits occur at precise energies, when these electrons jump from orbit to orbit the photons they absorb or emit to do so will be of very specific frequencies. This means the differences we see in the spectral lines of the different sources is due to the different molecules present and their specific electron orbits.

As each element has a unique spectrum, we can observe the spectrum of an unknown source and compare it to known sources to identify which elements are present. You have just used this process of elemental identification to determine which elements are present in your various sources.

Discuss whether this process could be applicable in astronomy when trying to identify the elements present in stars.

6. Conclusions

Spectral lines from different sources containing different elements or molecules were ____________________ (different/the same)

Some applications of studying spectral lines are:

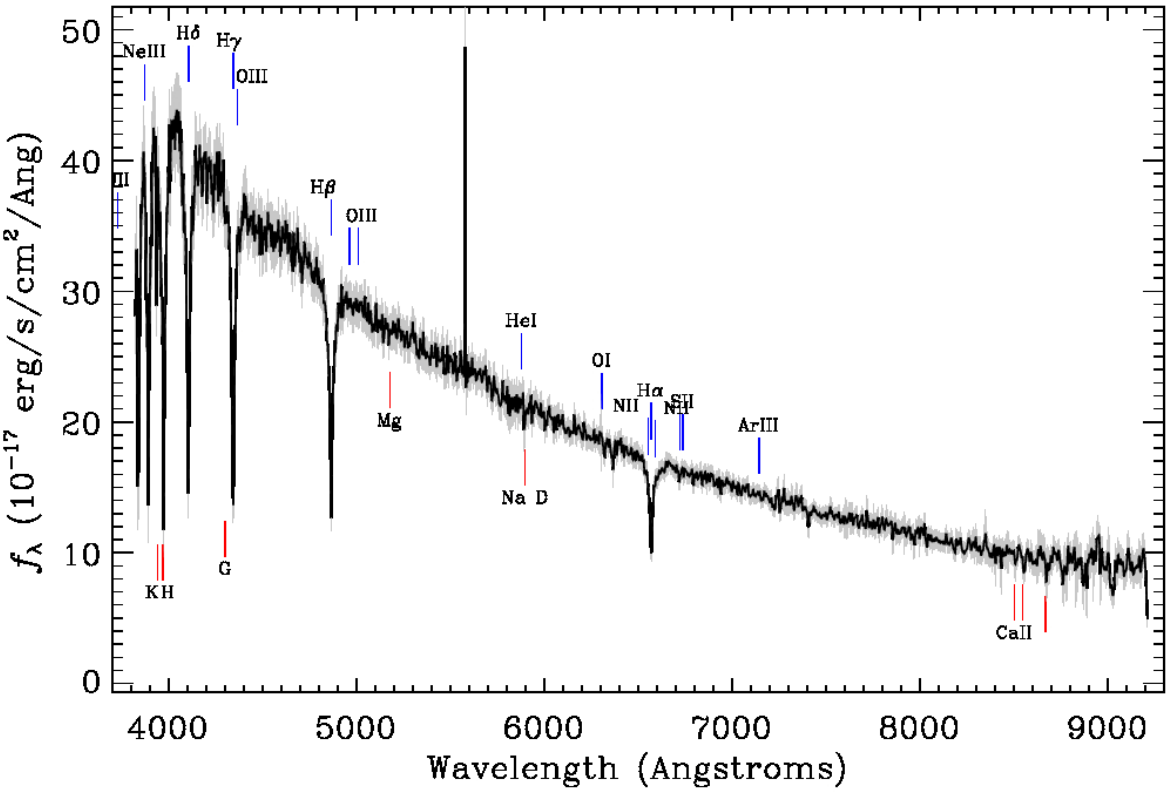
SPECTRA OF STARS

CONTENT

We have already discussed the use of spectral lines that we can observe from light sources, to identify the elements present in that source. However, we can also obtain the full spectra of astronomical objects like stars and obtain additional information including the surface temperature, rotational and translational velocity, density and chemical composition of a star.

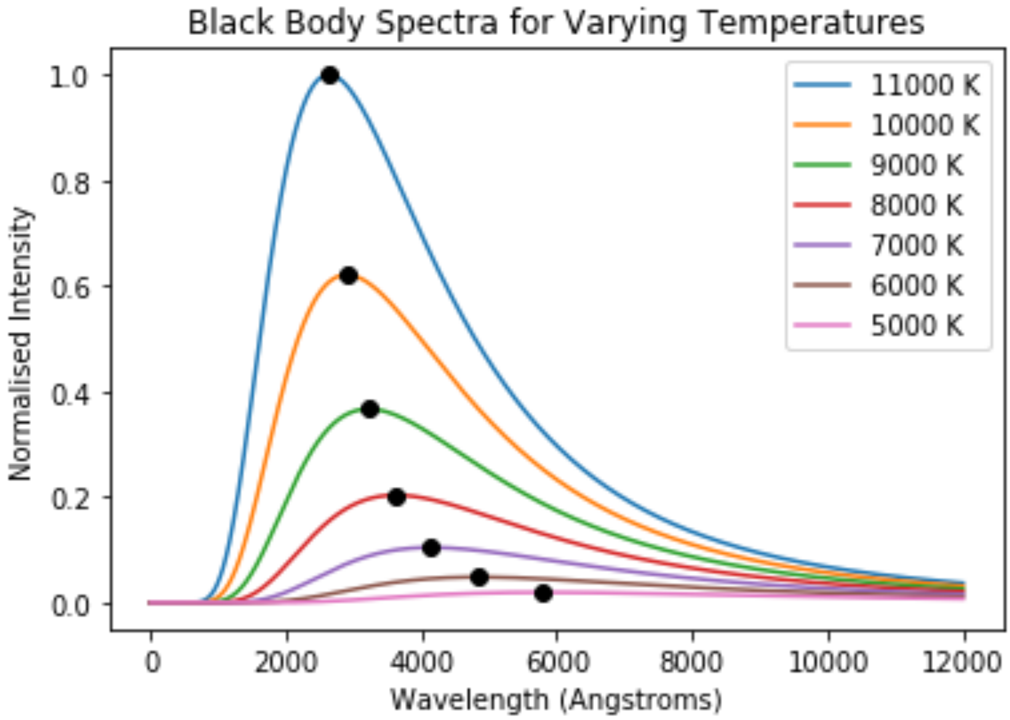
Before we extract all this information, we will first look at what the spectra of stars look like. The overall shape of these spectra follows a black body curve, which means that while they have a peak wavelength where they emit the highest intensity of light, they also emit light at other wavelengths. Here is a spectrum of a star from the Sloan Digital Sky Survey (SDSS) where we can see this overall black body envelope. We can also see various sharp peaks of absorption and emission lines due to the chemicals present in the stellar atmosphere.

We will now go through and discuss information we can gather about the star from its spectrum.



Surface Temperature

The peak wavelength of the spectra depends on the temperature of the star. This can be seen in the graph below where the black body curve is plotted for different temperatures and the maximum value for each curve is marked by the black dot. In this graph, we can see that the peak of the black body curves shifts to longer wavelengths for lower temperatures.



In order to calculate the surface temperature of the star we use Wien's Law

$$\lambda_{max} = \frac{b}{T_{surface}}$$

where λ_{max} is the peak wavelength of the black body curve in meters, $T_{surface}$ is the surface temperature of the star in Kelvin and b is Wien's Displacement Constant 2.898×10^{-3}Km. The peak temperature of stars is used to classify the spectral class of stars as O, B, A, F, G, K or M (O type stars have the hottest surface temperature and M the coolest), each of these classes can be broken down into 10 smaller classes, 1-10 for example A1 stars are hotter than A10 stars.

EXAMPLE 1

Going back to the spectrum of the star above, we can see there is a peak at around 4200Å. Calculate the surface temperature of this star.

⇒ Firstly, we will convert the wavelength from Angstroms to metres and then using Wien's Law we will calculate the surface temperature of the star. So:

$$\lambda = 4200\text{Å}$$
$$= 4.2 \times 10^{-7} m$$

$$\lambda_{max} = \frac{b}{T}$$
$$T = \frac{b}{\lambda_{max}}$$
$$= \frac{2.898 \times 10^{-3}}{4.2 \times 10^{-7}}$$
$$= 6900 K$$

Chemical Composition

When we look at the stellar spectra on the first page, we can see there are sharp drops in the brightness at very specific wavelengths. These drops are due to the absorption of very specific wavelengths of light by chemicals, called absorption lines. Each chemical element absorbs light at different wavelengths depending on the orbits and energies of the electrons in the atoms. For example, at 6563Å we see an absorption line. This is called the Hydrogen Alpha (Hα) line (it is labelled as such on the spectrum shown on page 1). Since we can calculate exactly which wavelengths we expect absorption lines to be at for each element, we can observe the spectra of stars and see which lines are present and figure out what elements are present in the star (you can check out the Spectroscopy Investigation to give this a go yourself).

We can also use these lines to determine if the star is moving (either towards or away from us), if it is rotating and also the density of the star.

Density

If the star has a low surface density (the gas of the out layer of the star is at a lower pressure), the spectral lines are sharper. The inverse is also true, if the star has a dense outer layer, meaning the gas is at a high pressure, the spectral lines are broader. While this is a similar effect to the broadening of spectral lines because of the rotational velocity (described below), they are subtly different allowing for astronomers to distinguish between the two.

Giant stars, like red supergiants, have a very low density and pressure in the outer layer. When we look at their spectra, we can see sharp spectral lines compared to white dwarfs which have a very dense outer layer. Comparing the spectra of the two allows astronomers to classify the Luminosity class of the star:

- Ia: Bright Supergiants
- Ib: Supergiants
- II: Bright Giants
- III: Giants
- IV: Sub Giants
- V: Main Sequence

- VI: Sub Dwarf
- VII: Dwarf

Our sun is a G2 V star because its surface temperature is quite cool (around 6,000K) and it is a main sequence star.

Rotational and Translational Velocity

Using spectral lines and the Doppler Effect, we can determine the different motions of the star, i.e. whether there is a translation velocity or rotational velocity. If you are unfamiliar with the Doppler Effect, you can revise it in the Doppler and Beats worksheets in Module 3.

Since chemicals absorb photons of very specific wavelengths (due to the structure of the atom or molecule) we see dramatic decreases in the intensity (or absorption lines) at those specific wavelengths. In the example above, we can see some have been identified. For example, the Hydrogen Alpha (Hα) absorption line is at 6563Å. When we put all this information together, we can identify what spectral lines are present and which elements are causing the absorption lines. We can also use the Doppler Effect to determine if the star is travelling towards or away from us (translational velocity) or is rotating on its own axis (rotational velocity). Both types of velocity have a measurable effect on the absorption lines in a spectrum.

Translational Velocity

If the star is moving towards or away from us, these absorption lines will be shifted to bluer and redder wavelengths respectively. So, we can compare the expected wavelength for a given absorption line (or the rest wavelength) with the observed wavelength and calculate if the star has any radial translational velocity. Similarly, we can calculate how quickly the star is moving. The amount of shift from the rest wavelength depends on how fast the object is moving. Using the equation from Year 11 below, we can calculate how fast the star is moving away from us

$$f' = f\frac{v_{wave} + v_{observer}}{v_{wave} - v_{source}}$$

where v_{wave} is the velocity of the wave, $v_{observer}$ is the velocity of the observer, v_{source} is the velocity of the source emitting the waves, f' is the observed frequency and f is the original frequency emitted.

It is important to note, we only see this effect if some component of the velocity of the star is directly towards or away from us. If the star is moving perpendicular to our line of sight (the imaginary line from the observer to the star), we see no effect on the absorption lines. This means we can only use the spectra to measure the line of sight velocity (called the radial velocity) rather than the total three-dimensional velocity.

Rotational Velocity

Just like with the translational velocity, if the star is rotating about its axis, the absorption lines will be shifted to redder or bluer wavelengths depending on the motion of the object emitting the light. When a star rotates, one side of the star will be moving towards the observer while the other side will be moving away. When we record the spectrum of a star, we observe one spectrum for the star. This combines the effect of the Doppler shift due to the rotation and as a result we see the same line shifted to redder and bluer wavelengths. Thus, the absorption line is actually blurred out and we see a broader emission line than we expect. Again, if the star is rotating very quickly, each side with shift the absorption line more, this means the broader the absorption line, the faster the star is rotating.

EXAMPLE 2

An astronomer is studying the spectra of a star and calculated the Hα absorption line at a wavelength of 656.4nm. However, the expected wavelength for the Hα line is 656.3nm. Given the speed of light is 3×10^8m/s and assuming the observer was not moving when the spectrum was taken, what is the radial velocity of the star?

⇒ Firstly, we will write down all the values we have and convert our wavelengths to frequency using $v = f\lambda$:

Variable	Value
$\lambda_{observed}$	656.4nm
$f_{observed}$	$4.57038 \ldots \times 10^{14}$Hz
$\lambda_{expected}$	656.3nm
$f_{expected}$	$4.57108 \ldots \times 10^{14}$Hz
$v_{observer}$	0m/s
v_{wave}	3×10^8m/s
v_{source}	?

⇒ Now, we will rearrange the Doppler Shift equation to make v_{source} the subject and then calculate:

$$f' = f\frac{V_{wave} + V_{obs}}{V_{wave} - V_{source}}$$

$$f'(V_{wave} - V_{source}) = f(V_{wave} + V_{obs})$$

$$V_{wave} - V_{source} = \frac{f}{f'}(V_{wave} + V_{obs})$$

$$\Rightarrow V_{source} = V_{wave} - \left(\frac{f}{f'}(V_{wave} + V_{obs})\right)$$

$$V_{source} = 3\times10^8 - \left(\frac{4.57108}{4.57038}\times10^{14}(3\times10^8)\right)$$

$$= 3\times10^8 - (1.00015\ldots\times3\times10^8)$$

$$= 3\times10^8 - 3.000457\ldots\times10^8$$

$$= -45.710\ldots\times10^3$$

$$\Rightarrow V_{source} = -45.71\ \text{kms}^{-1}$$

NEWTON VS HUYGENS

CONTENT

In the seventeenth century there was growing interest in the true nature of light; namely, whether it was a particle or a wave. Many experiments were done by different physicists in order to provide enough evidence for either case. Notably, Isaac Newton supported a particle model or corpuscular model while Christian Huygens argued a wave model. Both models had supporting evidence but were unable to completely explain the behaviour of light. At this time, it was already known that light travelled in straight lines, could reflect off surfaces and refract in mediums. In this worksheet we outline the experimental evidence for Newton's model and Huygens' model and then combine all the evidence to provide a conclusion.

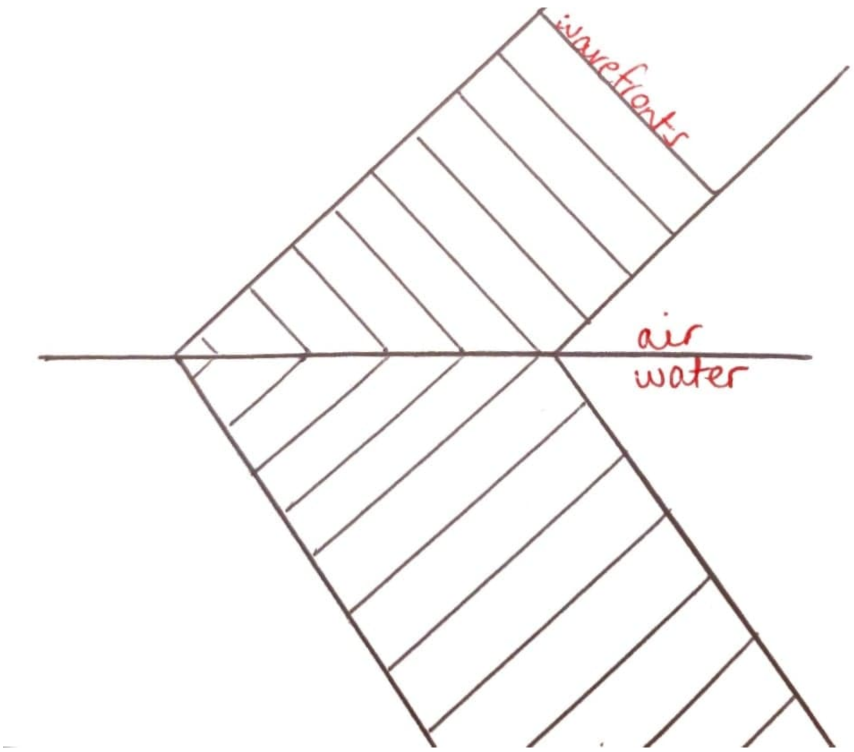
NEWTON'S MODEL

Newton supported the theory that light was made of tiny, rigid and massless particles known as corpuscles. These corpuscles were emitted in a constant stream from sources and travelled in straight lines away from the source. This can easily explain the observation that light travels in straight lines, it also does not require a medium for the particles to travel through which, as we will see in Huygens' model, avoids an added complication. Since these corpuscles are rigid massless particles, Newton was able to explain the reflection of light using the perfectly elastic collisions of the particles with a surface. This was also able to explain Snell's Law regarding the angles involved in reflections.

While Newton was able to explain the reflection of light and how it travels in straight lines using corpuscular theory, he had to introduce many complications to this theory to explain the refraction of light in a medium. Newton required a force of attraction between the corpuscles and the particles within the water that changes the direction of the particles. However, for this to be true, it requires light to travel faster in water than in air. On top of this, when Young performed his double slit experiment in 1801, Newton's model of light failed to explain these results at all. While the corpuscular theory of light was widely accepted from the 17th century (in part due to Newton's prestige over Huygens'), this result was the final nail in the coffin that led to a wave model being preferred.

Huygens' Model

Opposed to Newton was Huygens who proposed that light was a wave not a particle. However, at the time, it was believed waves required a medium in which to travel. Thus, Huygens also proposed an aether, an all pervasive, undetectable medium which allowed the light waves to travel through space. This caused a lot of doubt about Huygens' model and it was not readily accepted. Despite this, Huygens' model was able to explain reflection, refraction and, most notably, diffraction. Huygens proposed that sources emit spherical wavelets which travel outwards producing a wavefront. Waves allow the light to 'bend' around surfaces or diffract. This could explain the Fresnel Bright Spot, a bright dot observed in the middle of the shadow of a solid object. A wave model could easily explain how light is reflected off surfaces as this was already well known. But it also could simply explain refraction, compared to Newton's corpuscle model, and required that light travels slower in water, a fact we now know. We can see in the diagram below how the wavefront moves as it passes through the surface of water.



Conclusion

Both Huygens' and Newton's models have merit and are able to explain some properties of light, however neither is complete. With the addition of a quantum approach to light, our current model combines both a particle model and a wave model. However, the nature of the particles and the waves are slightly altered to the original models. We now know light as a wave is an electromagnetic wave, both an electric wave and a magnetic wave propagating together without the need for a medium. Similarly, we know photons are massless particles, however to explain diffraction we no longer need a complex force of attraction between particles.

SPECIAL RELATIVITY

CONTENT

We already know of Einstein's two postulates: 1) the speed of light in a vacuum is an absolute constant; 2) all inertial frames of reference are equivalent. From the second postulate, we can conclude that the laws of physics must also be the same in each inertial frame. In order for these two postulates to be true then other properties that we consider to be constant in Newtonian physics, such as mass, momentum, and time must not be constant when measured from different frames of reference. These concepts will be familiar to you from the old syllabus, however their description has changed somewhat in the new syllabus.

Under the old syllabus, and also in some older physics textbooks, it is said that the mass of an object increases with its speed:

$$m_v = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

In this equation, m_0 is the rest mass of the object, which is the mass of the object when the measurer is at rest relative to the mass. This is an intrinsic property of an object that does not depend on speed. This has very important implications for momentum, although the old syllabus did not include it. Using the definition of momentum as mass times velocity, we get

$$p_v = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

That is, the momentum of an object is greater than expected from Newtonian physics $p = m_0 v$, by the so-called Lorentz factor. If we were to calculate momentum using our traditional equation we find that between inertial frames the conservation of momentum is not always met, defying the second postulate. When velocities are small, i.e. $v \ll c$, this is a fine approximation but as velocities approach the speed of light this approximation no longer holds. This is also why it becomes harder and harder to accelerate an object as its speed approaches the speed of light.

In the new syllabus, and more generally in modern physics, we no longer use the first equation and we don't say that the mass of an object increases with speed but rather focus on the increase in momentum. In this more modern description the term 'mass' always refers to the rest mass. Hence, in the new syllabus (and in most textbooks), mass is given the symbol *m*, without the subscript and momentum is described as,

$$p_v = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where m is the rest mass, v is the velocity and c is the speed of light in a vacuum, roughly 2.998×10^8m/s. So the momentum of an object is greater than expected from Newtonian physics, and increases steeply towards infinity as the speed approaches c. For this reason, it becomes progressively harder to accelerate an object as it gets faster, and it is impossible to accelerate it up to the speed of light. However, when we calculate the total relativistic momentum in different inertial frames the conservation of momentum is not violated, as is the case when we calculate the total momentum in different inertial frames using the Newtonian equation.

N.B. This entire discussion does not apply to a particle with zero mass. According to Special Relativity, such a particle always travels at c and photons are the best-known example.

EXAMPLE 1

Calculate the relativistic momentum for a muon travelling at 0.99 times the speed of light and with a rest mass of 1.9×10^{-28}kg. Assume the speed of light is 3×10^{8}m/s.

⇒ Firstly, we write down all the variables we have:

Variable	Value
m	1.9×10^{-28}kg
v	$0.99c$
p	?

We will use the relativistic equation for momentum:

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

⇒ Now, to calculate the value of the relativistic momentum:

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1.9 \times 10^{-28} \times 0.99 \times 3 \times 10^{8}}{\sqrt{1 - \left(\frac{0.99c}{c}\right)^2}}$$

$$= \frac{5.643 \times 10^{-20}}{\sqrt{1 - 1.0199}}$$

$$= 4.0 \times 10^{-19} \text{kgms}^{-1}$$

EXAMPLE 2

Using the same values for velocity from Example 1, calculate the classical momentum of the muon and compare it with the calculated relativistic momentum in Example 1.

⇒ So, firstly, let us write down our variables and the equation we will use:

Variable	Value
m	?
p	4.0×10^{-19}kgm/s
v	$0.99c$

We will use the classical equation for momentum:

$$p = mv$$

⇒ Now, to calculate the mass:

$$p = mv$$

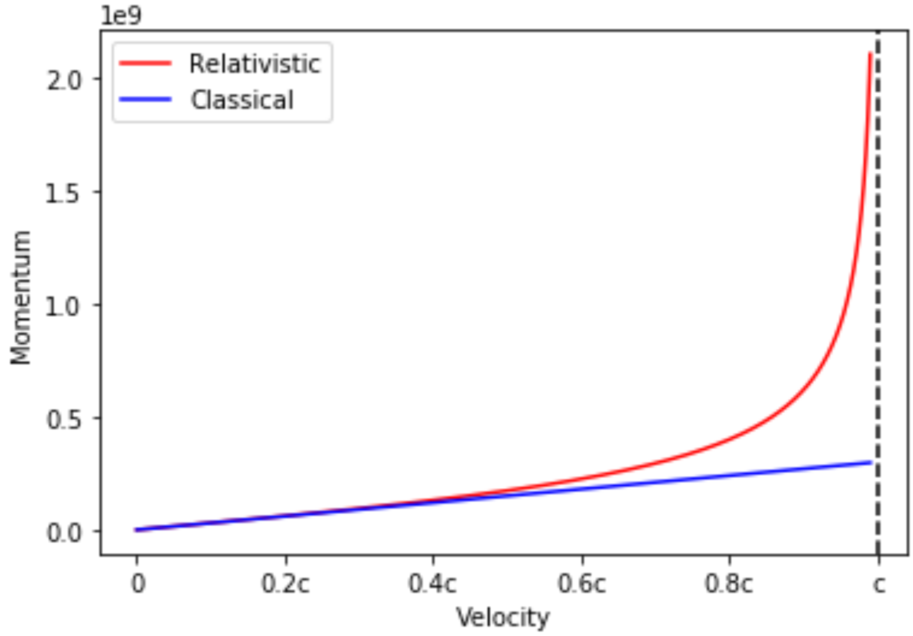
$$= 1.9 \times 10^{-28} \times 0.99 \times 3 \times 10^{8}$$

$$= 5.6 \times 10^{-20}$$

⇒ For the same mass, the classical momentum is less is than the relativistic momentum

EXAMPLE 3

Consider the graph below of the momentum for a particle with unit mass. Using both classical momentum and relativistic momentum justify the use of classical momentum for small velocities and relativistic momentum for velocities approaching the speed of light.



⇒ To tackle this question, we will consider it in two parts: smaller velocities below ~$0.5c$ and larger velocities above ~$0.5c$

⇒ Firstly, the smaller velocities. From the graph, we can see both classical and relativistic momentum are almost identical until around $0.5c$. This proves that while classical momentum doesn't account for relativity, it is a reasonable approximation for low velocities where the differences between the classical momentum and relativistic momentum are small.

⇒ However, for the second case (larger velocities above $0.5c$), we can see these two cases produce significantly different values for momentum, suggesting that the approximation of classical is no longer applicable at large velocities. As we approach the speed of light, c, it is only the relativistic momentum that correctly accounts for the maximum velocity imposed on a particle by special relativity. This is seen by the dramatic rise of relativistic momentum as we approach the asymptote at $x = c$. Classical momentum continues straight and would continue beyond the speed of light despite it being a maximum value; this is not a physical situation and thus only relativistic momentum is correct for large velocities and can account for velocities approaching c

ENERGY MASS EQUIVALENCE, E=MC^2

CONTENT

One of the best-known results from Einstein's Special Relativity Theory is the equation that relates energy and mass:

$$E = mc^2$$

where E is the energy, m is mass and c is the speed of light in a vacuum, 3.00×10^8ms^{-1}. This result has many applications and is able to explain where energy and mass go in certain reactions without violating the law of conservation of energy. Since the speed of light is a **very** large number, and in this equation it is being squared to make an even larger number, $E = mc^2$ shows how a very small amount of mass can be converted to produce a huge amount of energy. This had many implications, from explaining the energy of the sun, to where the energy in the combustion of conventional fuels and particle-anti-particle annihilations came from.

For many years, scientists had tried to explain where the energy of the sun came from. Calculations suggested it could only produce the energy it is currently producing for a period of at most a few million years. However, scientists had already aged rocks on the Earth to be over 4 billion years old. The energy mass equivalence explains where the energy from the sun comes from and how it is able to sustain itself for many billions of years.

Inside the sun, a process of nuclear fusion is occurring, a process that fuses, or combines, multiple atoms together to form a new product. The sun is fusing types of Hydrogen into Helium through several complicated reactions. However, the end product of this reaction has less mass than the beginning products. Using $E = mc^2$ we now know this missing mass is converted into energy.

Just like the nuclear fusion process in the sun, when a particle and its corresponding anti-particle collide, they annihilate each other and their mass is turned into energy via $E = mc^2$. If we know the mass of the two particles before the collision and there are no particles left after, we know all the mass has been converted to energy.

EXAMPLE

The mass of an electron is about 9.11×10^{-31}kg and a positron (its anti-particle counterpart) has the same mass. If an electron and positron collide and no particle is left after, how much energy is released by the conversion of mass to energy in the collision?

⇒ Firstly, we add the mass of the electron and the positron since the total mass converted to energy is the combined mass of the two. So, our m is 1.822×10^{-30}kg

⇒ Now to sub our values into $E = mc^2$ and calculate the energy released by the conversion of mass:

$$\begin{aligned} E &= mc^2 \\ &= 1.822 \times 10^{-30} \times (3 \times 10^8)^2 \\ &= 1.64 \times 10^{-13}\text{ J} \quad (3 \text{ sig fig}) \end{aligned}$$

⇒ Therefore, when all the mass of an electron and positron annihilation is converted to energy, 1.64×10^{-13}J are released.