EMISSION AND ABSORPTION SPECTRA

BLACK BODY RADIATION

Black-body radiation is the name given to the electromagnetic radiation emitted by an idealised opaque and non-reflective object, which is in thermal equilibrium with its surroundings. Put simply, a black body does not **reflect** any of the light which hits it, and all of the light which hits it is **absorbed**. However, despite its name a black body is still able to **emit** radiation. Approximating stars as black bodies is generally a reasonable approximation. Even though stars do reflect light the amount of reflected light is many orders of magnitude lower than that emitted due to black body radiation. A comparison between the observed spectrum of our sun, and a theoretical black body of the same temperature is shown in Figure 1. The overall shape is a good approximation; however, the solar spectrum has significantly more visible and infrared light, and less UV light.

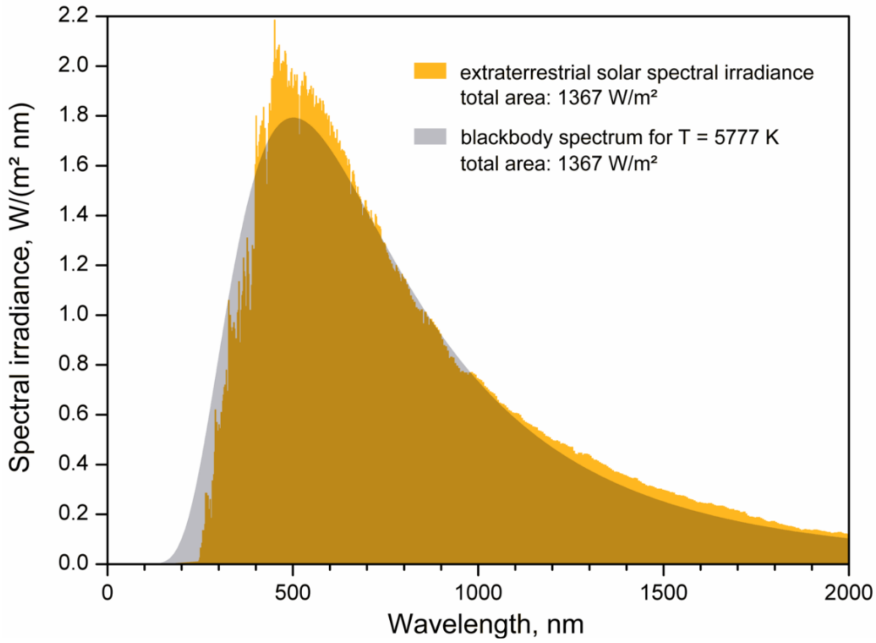

Figure 1. Comparison of observed solar spectrum and theoretical black body radiation
Image Credit: User:Sch/Wikimedia Commons/CC BY-SA 3.0

The wavelengths of the black body radiation are determined solely by the temperature of the material and not the composition of the material. Hotter temperatures shift the distribution to shorter wavelengths (bluer in colour) and colder temperatures produce longer wavelengths (redder in colour). Examples of these black body emissions for a range of temperatures are shown in Figure 2.

Another requirement for the black body radiation description to be valid is that the material must have a sufficiently large number of particles. Recall that there are discrete energy levels electrons can occupy in individual atoms. As we join atoms together the number of possible energy levels increases, and in the limit of a very large number of particles the number of energy levels grows to a point where it is no longer discrete as in a single atom but these possible energy levels become essentially continuous. This is why we get a seemingly continuous distribution in blackbody radiation. This is contrasted with the spectra we observe for gases, where the atoms are far enough apart that they maintain their discrete electron energy level structure. The emitted radiation from such a diffuse material appears as discrete 'spectral lines', which correspond directly to the discrete energy levels (this is covered in more detail in the resource 'Quantised Energy Levels').

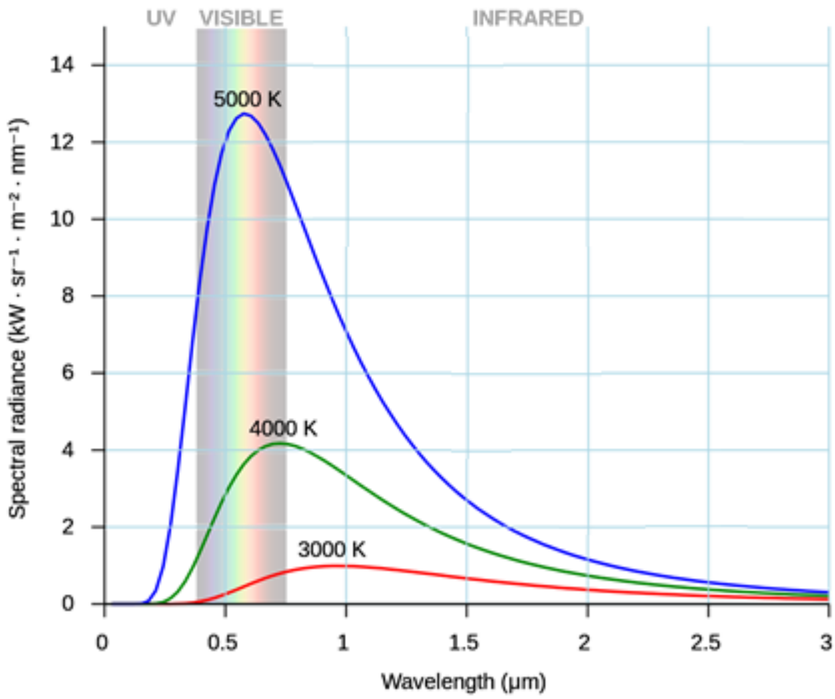

Figure 2. Black body radiation distributions for three different temperatures

Absorption and Emission

Although the bulk of a star is dense enough to produce continuous black body radiation, the outer layers are diffuse enough that the atoms in these outer layers maintain their discrete electron energy level structure. The electrons in these atoms can absorb specific wavelengths of light which correspond to these energy levels. This process causes absorption lines to appear superimposed on the continuous black body spectrum.

The inverse of absorption spectra are emission spectra. These spectra are observed when a diffuse region of gas (e.g. the outer layers of massive stars, or a galactic nebula) is excited by light from a star and they selectively re-emit this energy at the characteristic wavelengths of that gas. These emission spectra are found by observing the gaseous object from a direction which is out of line with the background source that is exciting the gas. These processes are depicted diagrammatically in Figure 3. Emission spectra can be used to determine the composition of gas clouds between the stars and the Earth.

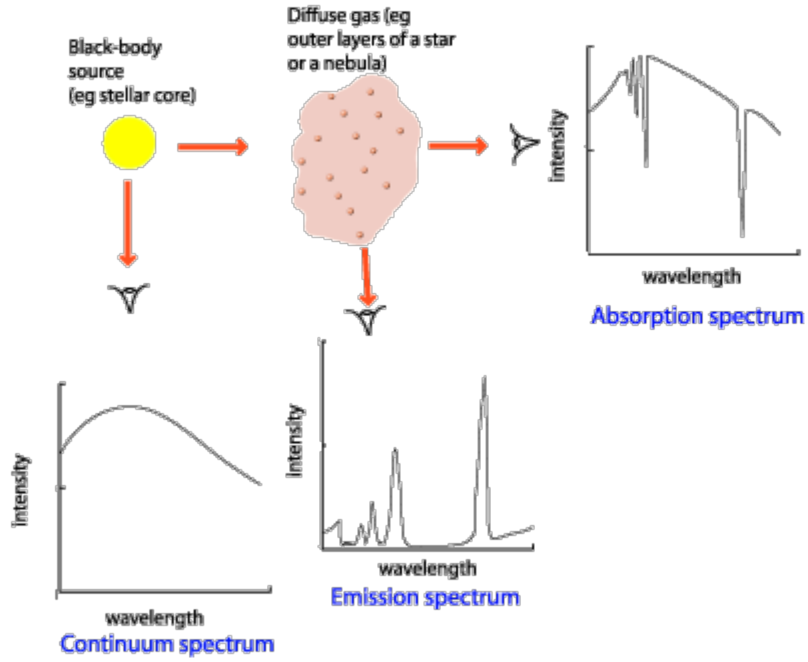

Figure 3. Creation of emission and absorption spectra
Image Credit: http://www.atnf.csiro.au/outreach/education/senior/astrophysics/

A real stellar spectrum is shown below in Figure 4. The most apparent or 'strongest' absorption lines correspond to the visible wavelengths of the Hydrogen atom known as the Balmer series.

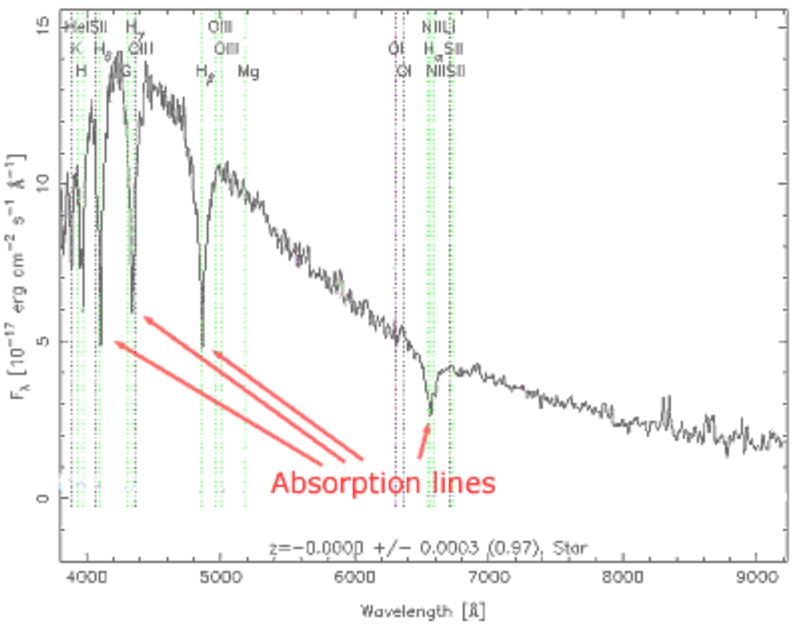

Figure 4. Real spectrum of a typical 'main sequence' star
Image Credit: The Sloan Digital Sky Survey

We can use these stellar spectra to classify stars by their temperature. By temperature we actually mean the 'effective temperature' of the star which corresponds to the temperature of the outer surface of the star that we actually observe. The inner core of the star is much hotter than the surface. The 'effective temperature' is determined by comparing the peak of the spectrum with the peak of the theoretical black body spectrum. Most stars can be classified into the following spectral classes, listed from hottest to coldest are O, B, A, F, G, K, M. The typical spectral features of these star are summarised in the table below.

Spectral Class	**Effective Temperature (K)**	**Colour**	**H Balmer Features**	**Other Features**	**M/M_{Sun}**	**L/L_{Sun}**
O	28,000 - 50,000	Blue	weak	ionised He^+ lines, strong UV continuum	20 - 60	90,000 - 800,000
B	10,000 - 28,000	Blue-white	medium	neutral He lines	3 - 18	95 - 52,000
A	7,500 - 10,000	White	strong	strong H lines, ionised metal lines	2.0 - 3.0	8 -55
F	6,000 - 7,500	White-yellow	medium	weak ionised Ca^+	1.1 - 1.6	2.0 - 6.5
G	4,900 - 6,000	Yellow	weak	ionised Ca^+, metal lines	0.85 - 1.1	0.66 - 1.5
K	3,500 - 4,900	Orange	very weak	Ca^+, Fe, strong molecules, CH, CN	0.65 - 0.85	0.10 - 0.42
M	2,000 - 3,500	Red	very weak	molecular lines, eg TiO, neutral metals	0.08 - 0.05	0.001 - 0.08

Table 1. Spectral features of stellar categories
Credit: Table reproduced from http://www.atnf.csiro.au/outreach//education/senior/astrophysics)

There are also rarer classes of stars that are distinguished by their relative abundance of carbon. For example R stars have the same temperature as K stars but have high abundances of carbon, and N stars are carbon-rich stars with the same temperature as M stars. Wolf-Rayet stars (classified as WN and WC), have the same temperature as O stars, but show strong broad emission lines of nitrogen or carbon. It is also important to mention that the light from distant stars often passes through diffuse gas clouds in between the star and earth. This can further complicate the interpretation of these absorption spectra. Aside from determining the chemical composition and temperature of stars, we can also use absorption spectra to infer the translational and rotational speed of a star. This is achieved by using the Doppler Effect and looking for the shift between the spectral lines observed in the stars and those from stationary sources here on earth.

HERTZSPRUNG RUSSELL DIAGRAM

CONTENT

The **Hertzsprung–Russell diagram**, or more simply **HR diagram,** was first created in 1910 by Ejnar Hertzsprung and Henry Norris Russell. It is a now famous scatter plot of stars which describes the relationship between the absolute magnitude or luminosity of stars and their stellar classifications or effective temperatures. More simply, each star is plotted on a graph with the star's brightness on the vertical axis and its temperature or colour on the horizontal axis. While there are several versions of the HR diagram, they all share the same general layout, with more luminous stars towards the top of the diagram, and fainter stars towards the bottom. Stars with higher surface temperature sit toward the left side of the diagram, and those with lower surface temperature sit to the right side of the diagram. Since stars are essentially hot black bodies, their colour is directly related to their surface temperature, with blue stars being the hottest and red stars being the coolest. The concept of blackbody radiation is covered in more detail in a previous resource. A descriptive version of the HR diagram is shown in Figure 1.

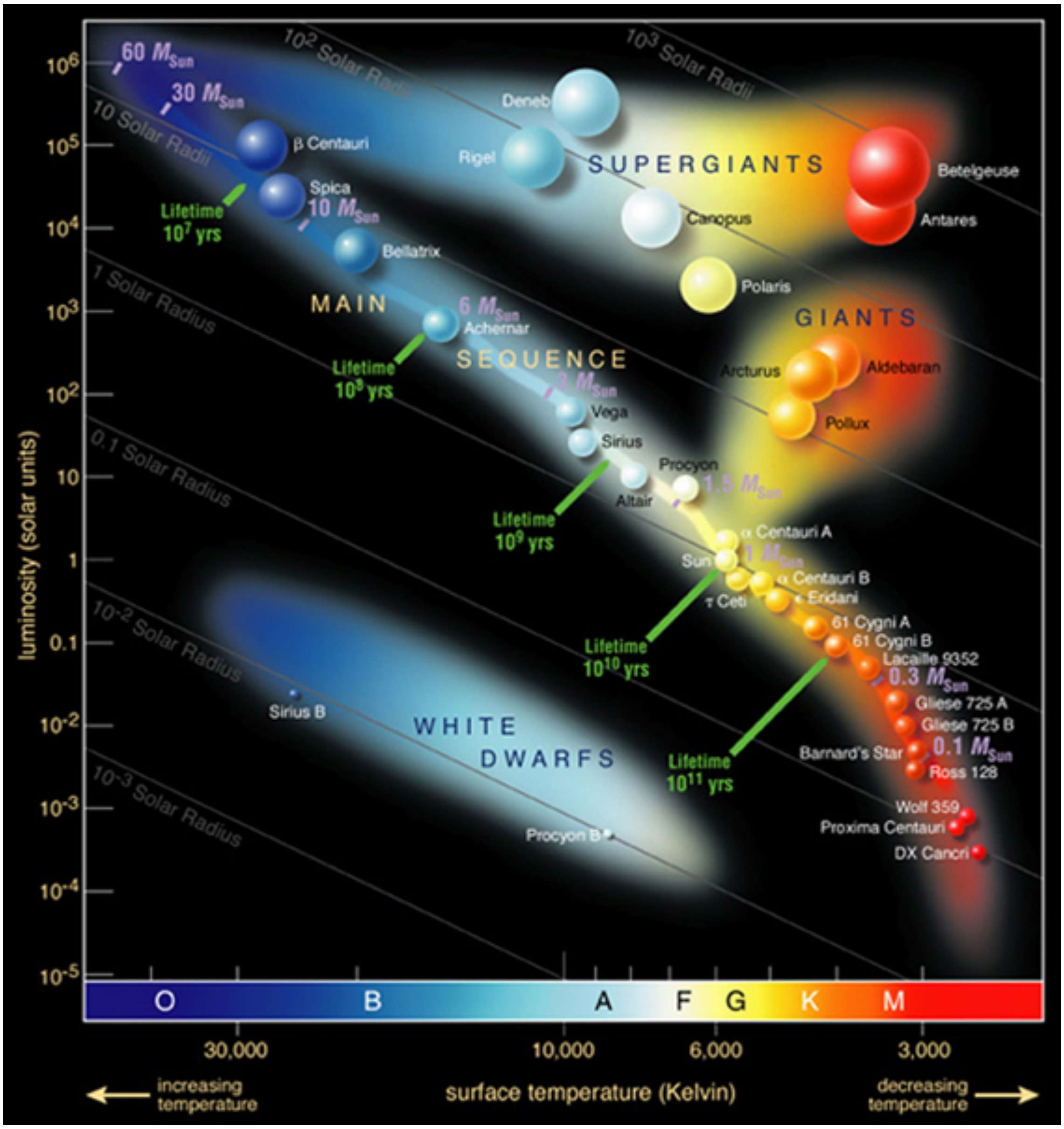

Figure 1: The Hertzsprung-Russell diagram
Image Credit: European Southern Observatory/**CC BY 4.0**

The life cycle of stars can also be tracked through the HR diagram. The basic stellar evolution of both high and low mass stars is covered in Figure 2. All stars begin as a star forming nebula of gas, and when enough mass has accumulated hydrogen fusion begins. The remains of the nebula that has not coalesced into a star is blown away by the energy generated by fusion. Once the fusion process has begun, the protostar is "born". All stars, regardless of their mass, begin their life on the "main sequence" (this grouping of stars can be seen in Figure 1). Very low mass stars about 1/10 the mass of our sun, which can be found in the bottom right corner of the HR diagram, are only massive enough to fuse hydrogen into helium. Once their hydrogen supply is close to running out they move to the left of the HR diagram and become blue dwarfs, and once their fuel is consumed entirely they will slowly cool, eventually becoming white and then black dwarfs. There are no blue dwarfs yet as the lifetime of these small stars is very long, and there has not been enough time to exhaust their fuel given the limited age of the universe. All white dwarfs that currently exist are the remnants of mid-sized stars such as the sun.

Mid-size stars, such as our sun, begin fusing hydrogen on the "main-sequence" but are massive enough to fuse their helium through the CNO cycle (covered in more details in the 'Origin of the Elements' resource). Once the helium burning stage begins, these stars grow significantly in radius and move vertically up the HR diagram as they get much brighter. The outer surface temperature also decreases and the stars become redder in colour. This is due to the larger size of the star, as the surface is now further from the core where most of the energy and heat is being created. These stars are now off the "main-sequence" and in the "giant" area. This cooler surface temperature gives these stars the commonly used name "red giant". Once the helium is exhausted the star collapses, creating a planetary nebula and ultimately a white dwarf. White dwarfs are located in the lower left of the HR diagram. These stars are small in radius and low luminosity as much of their mass is lost to the planetary nebula. However, they are still quite hot/blue when they are first created. They slowly move in a diagonal right-down direction as they slowly cool and become dimmer, eventually becoming so cool and dim that we can no longer observe them (black dwarfs).

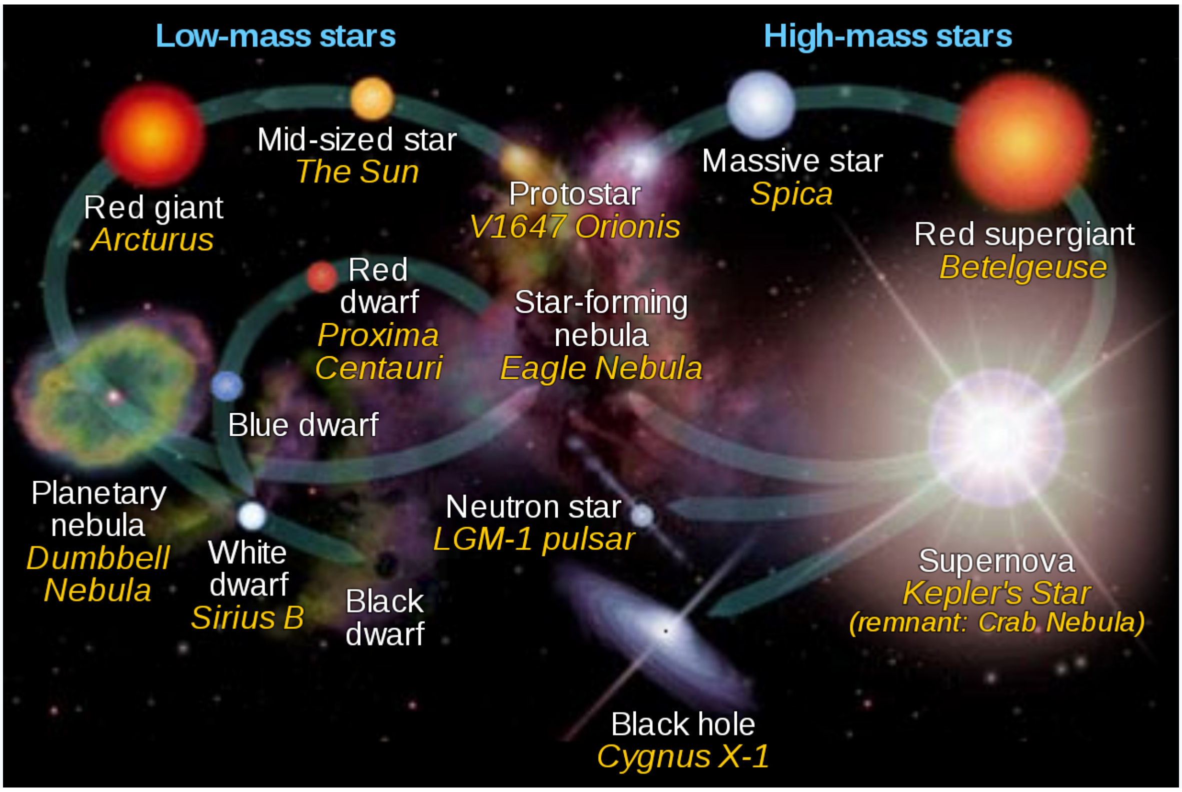

Figure 2: Life cycles of low and high mass stars
Image Credit: NASA Goddard Space Flight Center/**CC BY-SA 4.0**

High mass stars (>10 x mass of the sun) also begin on the "main sequence" as they consume their hydrogen. These stars quickly initiate helium fusion after they have exhausted their hydrogen, and are so massive that they will be able to continue fusing heavier elements after the helium fuel is exhausted. These stars will even fuse heavier elements up to iron until they develop an iron core. While these stars fuse these heavier elements they move off the main sequence towards the right. However, the most massive of these stars often don't make it as far right as less massive supergiants. This is because they are so energetic and massive that their outer layers are completely blown away before the star goes supernova. A supernova occurs once the fuel begins to run out and the gravitational pressure on the iron core becomes so large that the matter reaches a threshold where the repulsive force between the electron clouds that keeps atoms separate forces a collapse into the atoms' nuclei to form what is essentially one huge atomic nucleus made up of neutrons. We call this a neutron star. If the mass of the star is large enough then in a similar way the neutrons can also collapse and become a black hole, although this exact process is not well understood. These final states of the supergiant stars do not appear on the HR diagram.

The HR diagram can also be used by scientists to roughly measure the distances from Earth to a particular star cluster. This is achieved by comparing the apparent magnitudes of the unknown cluster stars to the absolute magnitudes of stars with known distances. The observed cluster is then shifted in the vertical direction on the HR diagram until the main sequence of the cluster overlaps with the main sequence of the reference stars of known distances. This technique is known as main sequence fitting, and by comparing the difference in luminosity between the unknown cluster and the reference stars, the distance to the unknown cluster can be determined.

ORIGINS OF THE ELEMENTS

CONTENT

The most famous equation in all of physics is, probably, Einstein's description of the equivalence of energy and mass. This relationship tells us that energy can be converted into mass and vice versa.

$$E = mc^2$$

The first thing that we notice from this equation is that due to the large value of the speed of light (c), it takes a lot of energy to create a small amount of mass. For example, the amount of energy equivalent in a cup (250mL) of water is 2.24x10^9 MJ. This is enough energy to power around 41,000 homes for 1 whole year! Let's take a look at how we calculated these numbers.

EXAMPLE 1

To demonstrate how to use this relationship we will calculate how much energy is equivalent to 250 mL of water.

First we need to know the mass of a cup of water. Given the density of water is 1000 kg/m^3, we can calculate the mass with the following equation:

$$\begin{aligned} mass &= density(kg/m^3) \times volume(m^3) \\ &= 1000 \times 0.00025 \\ &= 0.25\ kg \end{aligned}$$

Now that we have the mass of the water, we can calculate the equivalent amount of energy:

$$\begin{aligned} E &= mc^2 \\ &= 0.25\ \text{kg} * (2.998 \times 10^8\ \text{m/s})^2 \\ &= 2.247 \times 10^{16}\text{J} \\ &= 2.25 \times 10^{10}\ \text{MJ} \end{aligned}$$

This gives us the energy in Joules. However, we mostly measure electrical power consumption in units of kWh. We can use the conversion factor of 1MJ = 0.2778 kWh, thus

$$\begin{aligned} E_{kWh} &= 2.25 \times 10^{10} \times 0.2778 \\ &= 6.25 \times 10^9\ \text{kWh} \end{aligned}$$

The average Australian household consumes 40 kWh per day, meaning that the total number of houses that could be powered for a year would be:

$$Houses = \frac{6.25 \times 10^8}{40 \times 365} = 428{,}116$$

NUCLEAR FUSION REACTIONS

It is the conversion of mass to energy that is responsible for the enormous energy released by stars. Nuclear reactions known as fusion reactions are responsible for this release of energy. There are two major fusion reactions that take place in stars.

In each example below the mass of the helium nucleus produced is slightly less than the mass of the four hydrogen nuclei consumed to produce it. Slightly less than 1% of the mass involved in the reaction is converted to energy.

PROTON-PROTON FUSION

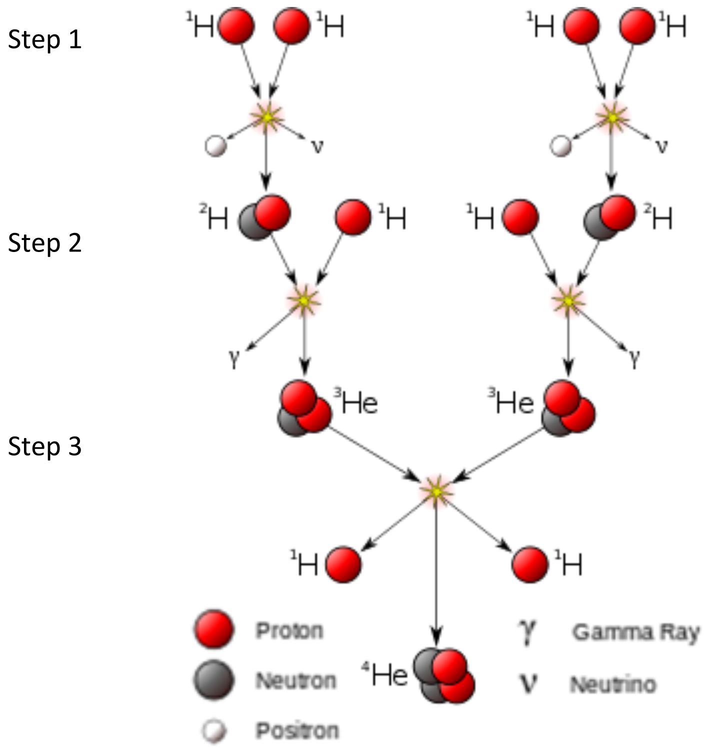

Figure 1. Schematic diagram of the proton-proton cycles
Image credit: User: Borb/Wikimedia Commons/CC BY-SA 3.0

$$4H \rightarrow He_4$$

Step 1: Two hydrogen nuclei, each a single proton, fuse together to create deuterium. This fusion process results in the emission of a positron (e+) and a neutrino.

Step 2: The newly-formed deuterium nucleus fuses with another hydrogen nucleus present inside the star. Together they form a ^{3}He nucleus, emitting a single photon.

Step 3: Two ^{3}He nuclei formed through the two steps above now fuse together, resulting in a stable ^{4}He nucleus and the release of two hydrogen nuclei back into the star.

This entire fusion cycle releases around 26.2 MeV of energy each time. This proton-proton cycle is the dominant source of energy in a main sequence star like our Sun.

CARBON-NITROGEN-OXYGEN FUSION (CNO)

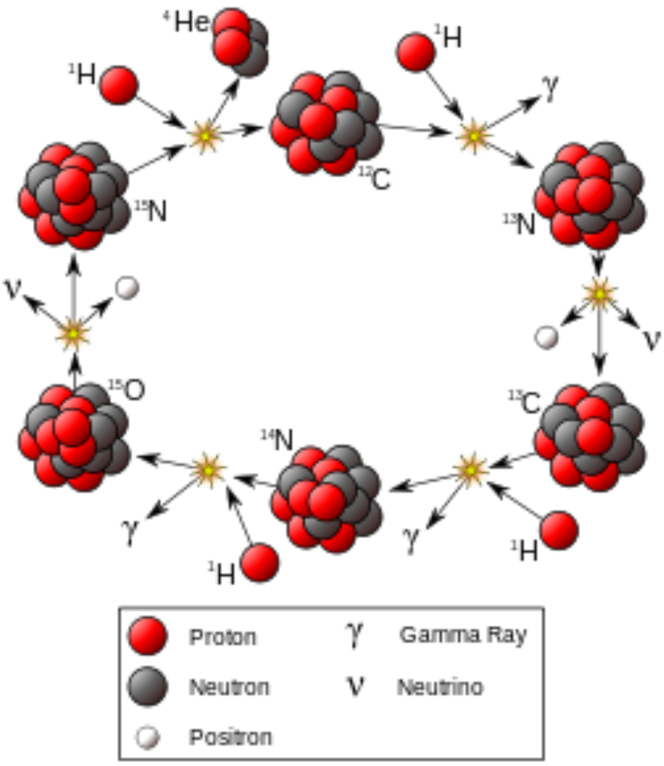

Figure 2. Shemcatic diagram of the CNO cycle
Image credit: User:Borb/Wikimedia Commons/**CC BY-SA 3.0**

Another way that stars can convert hydrogen into helium is through the carbon-nitrogen-oxygen (CNO) cycle. The CNO cycle is a catalytic process which involves multiple steps where heavier nuclei are involved in the reaction; however these nuclei are not used up in the process. Since the carbon nucleus is reconstructed at the end of the cycle, the overall reaction can be represented simply as $4H \rightarrow He_4$, as before. There are many slightly different versions of the CNO cycle, which are divided into two main categories of cold and hot CNO cycles, with the former being limited by the rate of initial proton captures, and the latter being limited by the rate of beta decays of the intermediate nuclei. An example of a cold CNO cycle is given below. This is the most common CNO cycle found in typical conditions within stars.

Step 1: A carbon-12 (^{12}C) nucleus fuses with a proton to form a nitrogen-13 (^{13}N) nucleus, emitting a photon in the process.

Step 2: The nitrogen nucleus releases a positron (e+) and a neutrino to become a carbon-13 (^{13}C) nucleus.

Step 3: The new carbon nucleus fuses with a proton to form nitrogen-14 (^{14}N), releasing a photon.

Step 4: The nitrogen nucleus fuses with a proton to form oxygen-15 (^{15}O), releasing a photon.

Step 5: The oxygen nucleus releases a positron and a neutrino to become a nitrogen-15 (^{15}N) nucleus.

Step 6: The nitrogen-15 (^{15}N) nucleus fuses with one final proton to produce a helium-4 (^{4}He) nucleus and a carbon-12 (^{12}C) nucleus that can be used once again for step 1, beginning the cycle over again.

Each complete CNO cycle produces about 25 MeV of energy. This is the main source of energy in higher-mass post-main sequence stars. This is because the repulsive electromagnetic force of the six-proton carbon nucleus prevents fusion without an incredibly high temperature and pressure to overcome it. Thus, it can only occur in the very centre of low mass stars, but is widespread in more massive stars. In the most massive stars even heavier elements, up to iron and nickel can form. Elements heavier than these typically don't form as iron and nickel are very stable and the star loses energy when fusing even heavier elements together. The heaviest elements tend to form a dense core in the centre of the star, with the lighter elements forming the outer layers of the star.

NUCLEAR MODEL OF THE ATOM

CONTENT

In the late 1800's and early 1900's there was a fierce debate about the composition of matter. While there were many ideas at the time, two models dominated the discourse. These were Thomson's 'plum-pudding' model, and Rutherford's 'atomic' model. The electron was discovered by J. J. Thomson in 1897 through his famous cathode ray experiment, and although this experiment wasn't conclusive, it suggested that the electron was a very light particle, about 2000 times lighter than the hydrogen atom. This result was conclusively determined by Millikan's oil drop experiment, where the exact charge of the electron was confirmed. This idea about a small and mobile electron was incorporated into both the Thomson model and the Rutherford model. Descriptive depictions of these models are shown in Figure 1.

Both models depicted the electrons in the same way (shown in blue), as small localised and mobile sources of negative charge. It was known that matter contains some amount of positive charge which balances out the negative charge of the electrons. It was also known that unlike the negative charge of electrons which are highly mobile, the positive charge contained within matter is locked in place. The key difference between the two models is the distribution of this positive charge. In Thomson's model the positive charge is spread out over a large volume, which makes up the bulk of the volume of matter. This can explain why the positive matter is not mobile as it is not condensed into small particles. Rutherford presumed that the positive matter should take on a much more similar form to the electrons. He thought that the positive charge should be localised as a discrete concentration of particles. Although much larger than the electrons, they still represented most of the mass of matter. This did suggest, however, that most of the space within matter is made up of nothing more than empty space. This is probably the most counter-intuitive part of Rutherford's model.

Definitive evidence for Rutherford's model came from the Geiger-Marsden experiment. A schematic diagram of this experiment is shown in Figure 1. An alpha particle source emits energetic particles in all directions. A collimated beam of alpha particles is created by encapsulating an alpha source in a box with a small hole in it. This beam is directed towards a very thin film of gold foil. Gold was chosen due to the fact that it is the most malleable metal and can be formed into incredibly thin sheets. The sheets had to be very thin as alpha particles have very poor penetration power and if the material was too thick then the alpha particles would be absorbed by the material. The expected results were quite different for the two models. Thomson's model predicted that due to the spread out nature of the positive charges the repulsive forces from the positive charge would be small and the deflection of the alpha particles would be minor. However, due to the high concentrations of positive matter in Rutherford's model, when an alpha particle approached one of these high concentrations of charge the predicted forces would be immense and the degree of deflection would be dramatic. The results were clear and clearly supported the 'atomic' model of the atom. After this discovery it was quickly surmised that the place of elements in the periodic table or 'atomic number' corresponded to the nuclear charge of the atom, and that the smallest atom (hydrogen) contained just one of these particles, which came to be named protons.

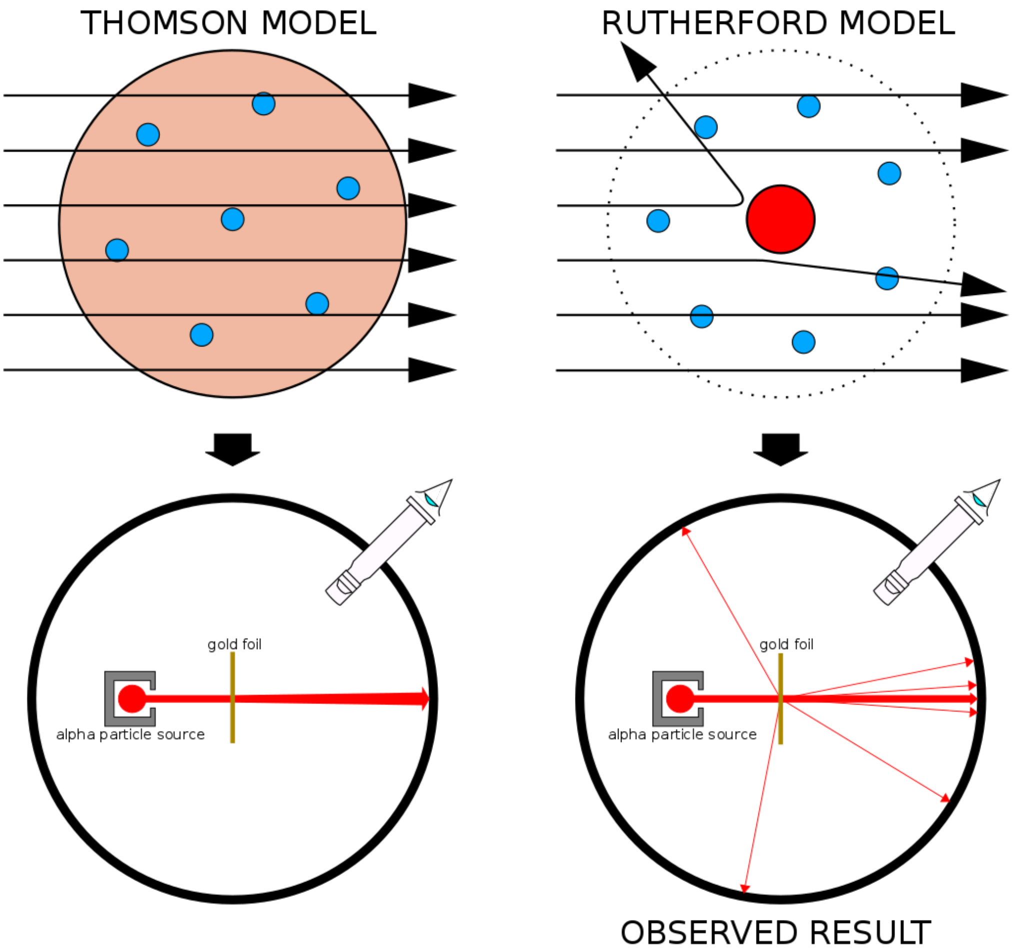

Figure 1. Comparison of Thomson and Rutherford models of the atom
Image credit: User:Kurzon/ Wikimedia Commons/CC BY-SA 3.0

After the validation of the atomic model by Rutherford, a more detailed version of Rutherford's model, called the Rutherford-Bohr model, or just the Bohr model was developed by Rutherford and Bohr. A schematic diagram of this model is shown in Figure 2. The core feature of this model, which differs to the earlier Rutherford model, is the postulated fact that the electrons are able to occupy certain stable orbits without emitting radiation (in conflict to classical electromagnetism). Electrons cannot occupy orbits between these stable orbits and the stationary orbits are located at distances which correspond to an electron angular momentum that is a multiple of the reduced Planck's constant ($n\hbar$). The final major point of the Bohr model is that electrons can move to different stable energy levels by the absorption and emission of electromagnetic radiation with a frequency that corresponds to the energy difference between the energy levels. This model was able to explain the atomic spectra of Hydrogen and similar atoms. This was a major improvement on previous models and suggested that there was some validity to the concept of 'allowed electron energy levels' in atomic structure. Despite these improvements, the Bohr model was not able to predict atomic spectra for the majority of atoms; it could not explain why some spectral lines were much more intense than others, it could not explain the Zeeman effect (where spectral lines are split in the presence of a magnetic fields, and most importantly it could not explain any reasoning as to why these energy levels were allowed and others were forbidden. It would take the introduction of quantum mechanics to answer these questions.

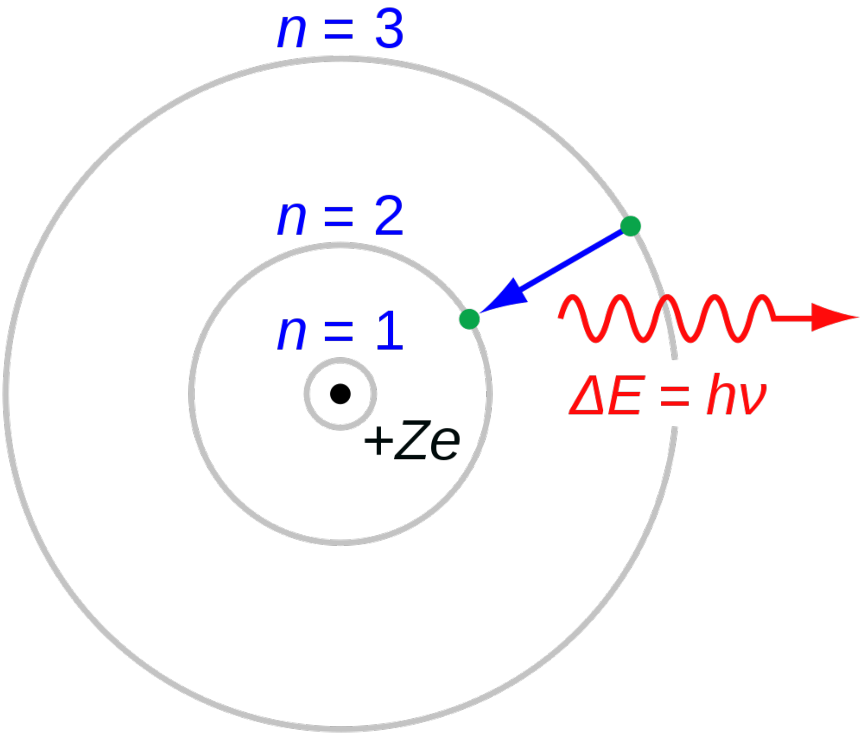

Figure 2: The Bohr model of the atom
Image credit: User:JabberWok/Wikimedia Commons/ CC BY-SA 3.0

The discovery of nuclear isotopes (that is, atoms that have the same atomic number and chemical nature, but a different atomic weight) was the first major piece of evidence that suggested the existence of the neutron. It was found that the mass of these isotopes could always be expressed as a whole number of proton masses, suggesting the existence of a particle that had the same mass as a proton but was neutral in terms of electric charge. Due to the fact that the neutron has no electric charge and the difficulty in creating free neutrons separate from the atomic nucleus, it would remain difficult to prove its existence.

An unknown and highly penetrating form of radiation was observed to be emitted when alpha particles struck specific light elements, such as beryllium. Initially, this radiation was thought to be gamma rays, but it was more highly penetrating than any gamma rays previously observed. It was also found that this unknown radiation caused the ejection of protons from paraffin wax at very high energies of 5 MeV. In order for gamma rays to be responsible for this interaction they must have an unbelievably high energy. James Chadwick was one of the physicists who did not believe the gamma ray explanation and designed the experiment shown in Figure 3 in order to study the reaction. By carefully measuring the range of the ejected protons and the interaction of the unknown radiation with a number of gases, Chadwick was able to determine that the unknown radiation could be interpreted as a neutral particle with about the same mass as the proton. Chadwick's discovery of the neutron provided the final piece in the atomic model puzzle and, in 1935, won him the Nobel Prize on Physics.

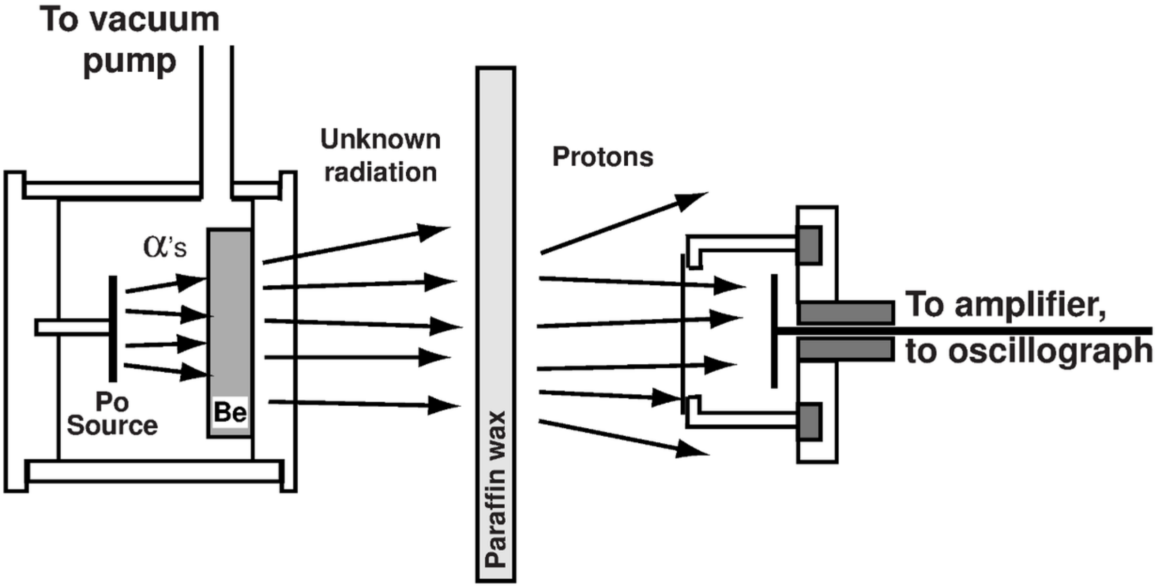

Figure 3: Schematic diagram of the experiment Chadwick used to discover the neutron
Image credit: User:Bdushaw/Wikimedia commons/CC BY-SA 4.0

ORIGINS OF THE UNIVERSE

CONTENT

Current scientific theory holds that at some point in history, there was no universe. In fact, time and space themselves didn't even exist before the explosion of radiation known as the 'Big Bang'. Suddenly, nearly 14 billion years ago, energy burst into being from a singularity and ultimately created everything that exists in our universe today, from atoms to stars to galaxies. But how did a bunch of light become physical matter?

Scientists believe that the universe was born at an incredibly high temperature, somewhere around 10^{32} K. With this much heat energy around, physical matter like particles and atoms couldn't exist, and the four fundamental forces of the universe we're familiar with - gravity, electromagnetism, and the strong and weak nuclear forces - were unified into a single superforce. We can trace a somewhat speculative timeline of the early universe as it expanded and cooled.

TIMELINE

- *Inflationary phase* (t = 10^{-43} to 10^{-35} s): The universe undergoes exponential expansion and cools down rapidly to ~10^{27} K. Gravity separates itself out as an independent force.

- *Age of leptons* (t = 10^{-35} to 10^{-6} s): The strong nuclear force separates, followed by the weak nuclear force and electromagnetism. Fundamental particles like quarks, leptons and photons come into existence.

- *Age of nucleons* (t = 10^{-6} to 225 s): Composite particles made up of two or more quarks known as hadrons come into being. Thanks to the high temperature, photons combine to form matter/anti-matter pairs, which then annihilate each other to create more photons in a thermal equilibrium. These processes are detailed in the equations following.

$$\gamma + \gamma \Leftrightarrow e^- + e^+$$

$$\gamma + \gamma \Leftrightarrow p + \bar{p}$$

$$\gamma + \gamma \Leftrightarrow n + \bar{n}$$

As this process continues, the universe cools down to 10^{11} K, too cold for continued proton and neutron anti-matter pairs. There is slightly more matter than antimatter overall.

- *Age of nucleosynthesis* (t = 225 s to 1,000 years): Protons and neutrons, collectively known as nucleons, begin to combine with each other to form nuclei which then react with each other to form bigger nuclei. By the end of this stage about 25% of the universe is helium, and the remainder mainly hydrogen.

- *Age of ions* (t = 1,000 to 3,000 years): The temperature of the universe is still high enough to ionize any atoms formed, so it's made mostly of photons, light nuclei and other charged particles.

- *Age of atoms* (t = 3,000 to 300,000 years): The temperature drops to 10^5 K, allowing nucleons and electrons to form stable atoms of hydrogen and helium. Following the production of stable atoms, photons no longer interact frequently with normal matter in the thermal equilibrium described above. This electromagnetic radiation spreads out to fill the universe. Today, this ancient radiation can still be detected as a faint, omnipresent glow known as the cosmic microwave background, with a temperature equivalent to about 3 K.

- *Age of stars and galaxies* (t = 300,000 years to present): Matter aggregates into clumps. Extremely dense clumps allow hydrogen atoms to undergo nuclear fusion, and create the first stars.

HUBBLE AND THE EXPANDING UNIVERSE

Up until the early 1900s most scientists assumed that the universe was overall static. This means that while it was thought possible that stars and galaxies could move relative to each other, it was thought that the space in which all objects exist in does not expand or contract in any way. However, since we know that gravity is an attractive force between all matter, why has the matter in the universe not coalesced into one gigantic mass? This question was pondered by many; in fact even Newton raised this as a serious issue with his theory of gravitation.

In the 1920s, the American astronomer Edwin Hubble conducted an investigation into the observations that other astronomers had made of the red-shift of distant 'nebulae' (now known to be galaxies) a known distance away – especially studies by astrophysicist Henrietta Leavitt and her observation of variable stars in the Small and Large Magellanic Clouds. It was observed that characteristic spectral lines of stars (covered in detail in the resource 'Stellar Spectra') were red-shifted for distant galaxies when compared with the wavelengths of these same spectral lines when measured on earth. It was initially assumed that these shifts were due to the Doppler Effect. This was a known process where the relative motion between a source and an observer causes a shift in the wavelengths measured. This process is depicted in Figure 1. However, we now know that this interpretation is not accurate.

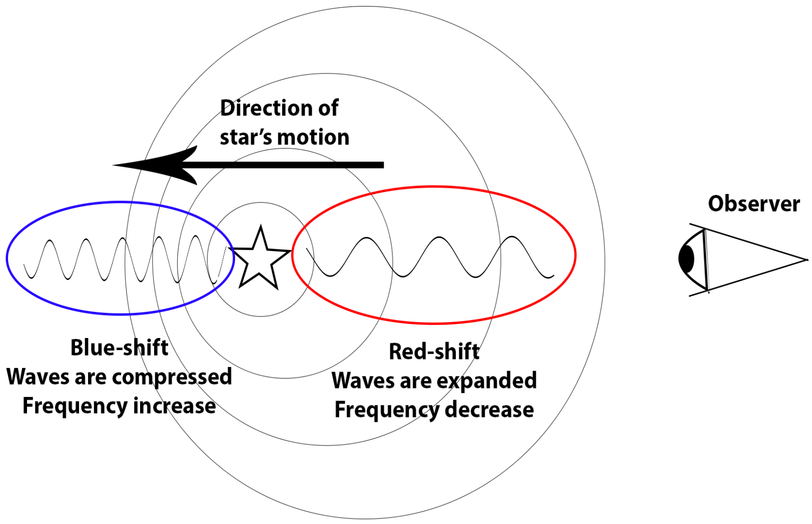

Figure 1. The initial interpretation of Hubble's Data was Red/blue shift from stars motion

Two remarkable results were found from analysing Hubble's data. First, it was found that the speed with which a galaxy is moving away from us is proportional to its distance, with the most distant galaxies moving the fastest and the closest galaxies moving the slowest. This relationship is called Hubble's Law. The second major discovery is that the light from *all* of the galaxies is 'redshifted' indicating that all galaxies are moving away from us. We have no reason to think that we are located at the centre of the universe, or that the region of space that we are located in is special in any way. This is a consequence of what is known as the cosmological principle, which states that at a large scale the universe is homogeneous and isotropic. This leads us to conclude that from any galaxy it appears that all other galaxies are moving away or that from any point in the universe it appears that the universe is expanding in all directions. Hubble's law supports the theory of a Big Bang, as it indicates that all of the matter in the universe was far more concentrated in the past than it is now.

Our understanding of the universe has improved with the introduction of the theory of general relativity. According to general relativity, the redshift in the observed wavelength of light is not caused by a Doppler shift as distant galaxies expand into a previously empty space. Rather, the observed redshift comes from the expansion of space itself and everything in intergalactic space, including the wavelengths of light traveling to us from distant sources. This can be a difficult concept to grasp at first. Consider the example shown in Figure 2, this shows a section of the 'universe'. Note that the relative position (grid-coordinates) of the galaxies remains the same, while the distance between the galaxies has expanded, and the wavelength of the light between the galaxies has also expanded. This is a direct effect of the fact that it is the space on which the grid, galaxies, and light are placed which is expanding. The picture below is a little misleading as it still shows a 'finite' universe which seems to expand into 'empty' space on the page. This is where this simple illustration breaks down, as our best current evidence suggests that the universe is infinite. This means that there is nothing outside the universe, it has no edges, and it is not expanding into anything. This is a hard concept to capture in an illustration!

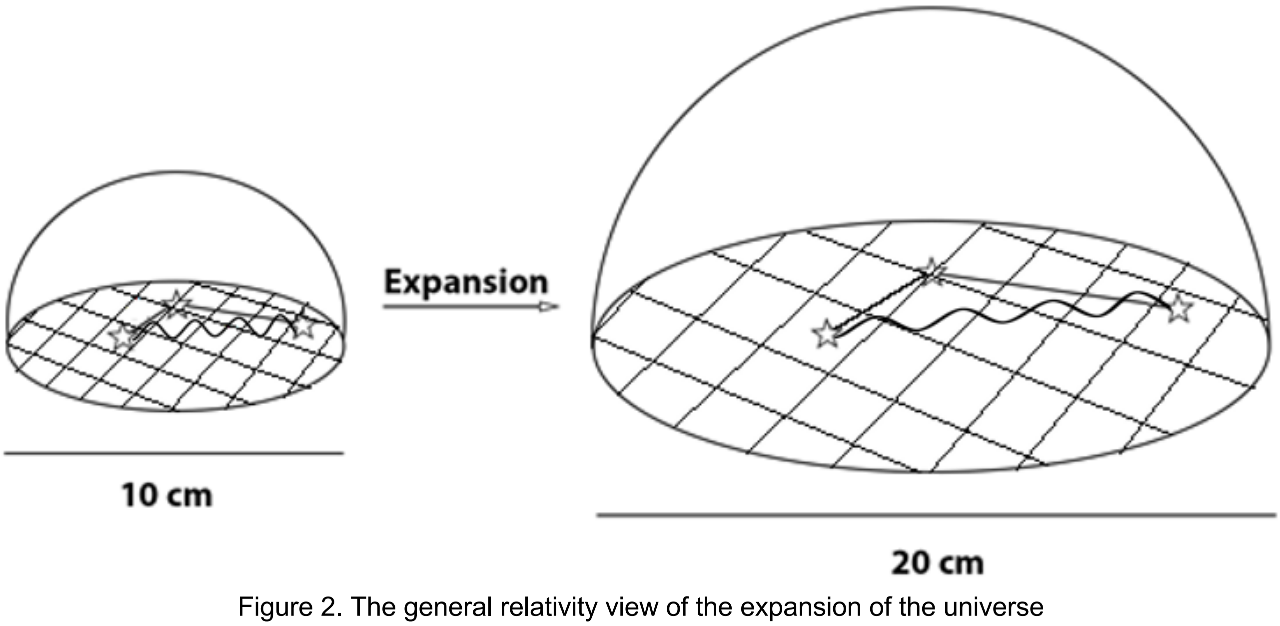

Figure 2. The general relativity view of the expansion of the universe

QUANTISED ENERGY LEVELS

CONTENT

The Balmer series refers to a series of visible spectral lines that are observed when hydrogen gas is excited. These lines are shown in Figure 1 (upper). The four most prominent lines occur at wavelengths of 410 nm, 434 nm, 486 nm, and 656 nm. The combination of these spectral lines is responsible for the familiar pinkish/purplish glow that we associate with hydrogen discharges and gaseous nebulae, shown in Figure 1 (lower).

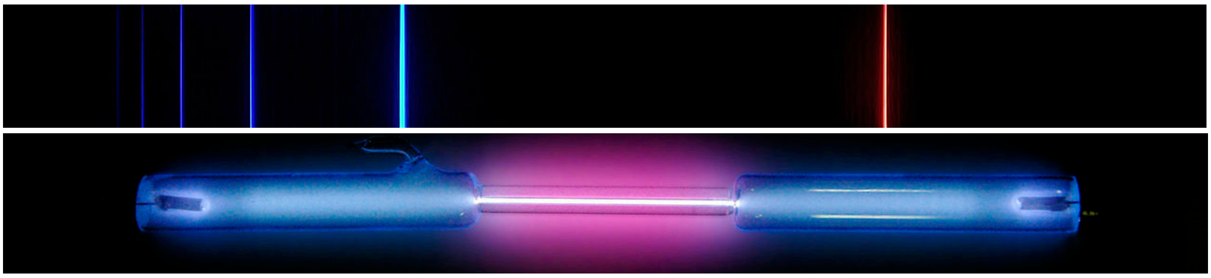

Figure1. The six lines of the Balmer series (upper), Hydrogen discharge (lower)
Image Credit: User: Jan Homann/Wikimedia Commons/CC BY-SA 3.0 & User: Alchemist-hp/(www.pse-mendelejew.de)/FAL

These spectral lines were observed long before an empirical equation was discovered by Johann Balmer in 1885 which could describe the wavelengths of the lines. However, it was not known at the time why this equation matched the observed wavelengths. Balmer's equation for wavelengths is given as

$$\lambda = B\left(\frac{n^2}{n^2 - m^2}\right),$$

where λ is the predicted wavelength, B is a constant with the value of 364.50682 nm, *m* is equal to 2, and *n* is an integer such that $n > m$.

We now know that this equation was able to describe the wavelengths of the Balmer series because these wavelengths correspond to electron transitions between higher energy levels and the n=2 energy level. As electrons transition from higher energy levels to lower energy levels, conservation of energy is maintained by the emission of a photon of energy equal to the difference between the final and initial energy levels of the electron. Such a transition is depicted in Figure 2, for the H_α line. We also know the difference between successive energy levels decreases as *n* increases. In addition, in the limit as n goes to infinity, the wavelength approaches the value of the constant B. This limit corresponds to the amount of energy required to ionise the hydrogen atom and eject the electron. This is also referred to as the electron binding energy.

As previously stated, the law of conservation of energy dictates that the frequency of the emitted photon be equal to the energy difference between the transitioning electron energy levels in the atom. This relation is quantified by the Einstein-Planck equation (sometimes referred to, in the context of electron transitions, as Bohr's frequency condition), given as

$$E = hf,$$

where E represents the difference in electron energy levels (this is also equal to the photon energy), *h* is Planck's constant (6.626 $\times 10^{-34}$ J.s), and *f* is the frequency of the emitted photon. We can also use the wave equation,

$$c = f\lambda,$$

where *c* is the speed of light, *f* is the frequency of the light, and λ is the wavelength of the light, in order to relate the energy of the electron transitions directly to the wavelength of the emitted photon

$$E = \frac{hc}{\lambda}.$$

Other hydrogen spectral series were also discovered, for instance the Lyman series describes transitions that end on the $n = 1$ energy level. As a result, the photons emitted from these transitions are higher in energy than the Balmer series, and the Lyman series is in the ultraviolet region of the electromagnetic spectrum. Another example is the Paschen series; this series describes transitions to the $n = 3$ level, so the emitted photons are lower in energy than the Balmer series and are categorised as infrared.

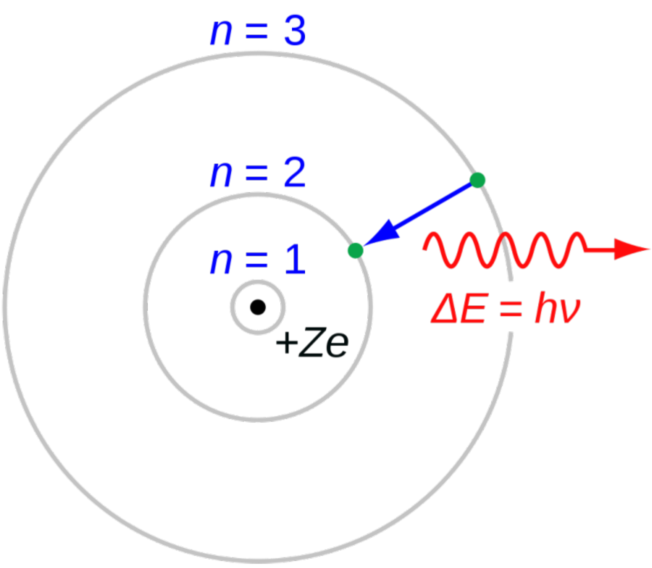

Figure 2: The Bohr model of the atom showing the H_α line
Image credit: User: JabberWok/Wikimedia Commons/ CC BY-SA 3.0

The Balmer equation was generalised by Rydberg in order to describe the wavelengths of all of the spectral series of hydrogen, including for instance the Paschen and Lyman series. The Rydberg equation is described as:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right),$$

where λ is the wavelength of electromagnetic radiation emitted in vacuum, R_H is the Rydberg constant for hydrogen with a value of $1.097 \times 10^{7} \text{m}^{-1}$, and n_f and n_i correspond to the final and initial energy levels of the transition.

Example 1:

Determine the wavelength of the 2nd line in the Lyman series (electron transition from n = 3 to n = 1), known as Lyman β

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{8}{9}\right)$$

$$\lambda = 1.0255 \times 10^{-7}\text{m}$$

$$\lambda = 103\text{ nm}$$

DE BROGLIE AND SCHRÖDINGER

CONTENT

Louis de Broglie was a French physicist who postulated that electrons could exhibit wave behaviour. This was based on the explanation Einstein gave for the photoelectric effect. Recall from the previous module 'The Nature of Light' that the photoelectric effect was an experimental result involving the interaction between light and metallic surfaces. These results could not be explained by the wave model of light but were explained successfully by Einstein using a particle model of light. He described the energy and momentum of the individual particles of light (photons) as:

$$E = hf$$
$$p = \frac{h}{\lambda}$$

In 1924, as a part of his PhD thesis, De Broglie postulated that this relationship between momentum and wavelength could apply to electrons as well. By rearranging the momentum equation, and substituting in the classical equation for momentum ($p = mv$), he was able to predict the wavelength of electrons as:

$$\lambda = \frac{h}{mv}$$

These waves are referred to as 'matter waves' or 'De Broglie waves', and this relationship is now known to hold true for all types of matter, not only electrons. This was verified experimentally for the first time in 1927 with the Davisson–Germer experiment. After this verification De Broglie won the Nobel prize in physics for his prediction. The Davisson–Germer experiment was based on the principle of diffraction. Diffraction is the general principle that occurs when waves (light or other) encounter an object or aperture (covered in the previous module 'The Nature of Light'). It was previously determined by Bragg that the crystal lattices of certain materials can cause diffraction patterns when X-rays pass through them. The Davisson–Germer experiment was simple in principle: an electron beam was fired at a nickel target and a detector was moved about in angle to search for reflected electrons, as shown in Figure 1.

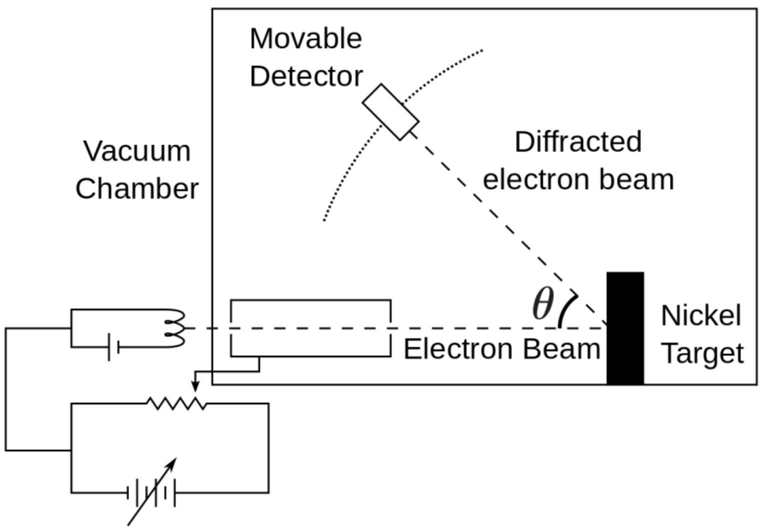

Figure 1. Experimental setup of the Davisson–Germer experiment
Image Credit: User:Roshan/Wikimedia Commons/ CC BY-SA 3.0

This experiment was not initially designed to look for diffraction patterns but for looking at how electrons are reflected from the surface of the material. One day when the nickel target was accidentally exposed to air, an oxide layer developed on its surface. In order to remove this oxide layer the nickel target was heated in a high temperature oven. While this removed the oxide surface, it also had the unintended effect of turning the polycrystalline structure of the nickel into much larger areas of consistent crystal structure. These consistent crystal areas were now large enough to cover the whole width of the electron beam. When the experiment was repeated with the new target, a diffraction pattern with unexpected peaks was observed, similar to those observed with x-rays by Bragg. The results were published by Davisson and Germer but they did not understand what to make of them. It was Max Born who saw the diffraction patterns and realised that they were in fact the first experimental confirmation of the De Broglie Hypothesis.

The idea of matter waves was extended by Erwin Schrödinger through a series of three papers published in 1926. In these papers he presented what is now known as the Schrödinger equation, which describes what we now know as 'wave-functions'. The wave function is a mathematical description of the state of a quantum system (e.g. electrons bound to atoms). It also describes the probabilities for the outcomes of making measurements of that system. These concepts formed the basis of quantum physics and have led to Schrödinger being referred to as the 'father of quantum physics'. Schrödinger also provided solutions to the Schrödinger equation that corresponded to the energy levels for hydrogen or hydrogen like atoms (atoms with only one electron). This model of the atom was able to correctly predict the spectral lines associated with these energy levels. This new quantum model was also able to account for other things that the previous model (Bohr model) of the atom could not, such as the various intensities of spectral lines, and the Zeeman Effect. We still use the Schrodinger equation and the concept of wave functions to describe atoms, and we now refer to the states of the electrons in atoms as atomic orbitals. An example of the atomic orbitals found in the hydrogen atom is shown in Figure 2. Note that unlike the electron orbits of earlier atomic models these orbitals take on many different shapes, not only spherical. Also, rather than having an electron as a discrete particle orbiting a nucleus, the atomic orbitals are more akin to a cloud which surrounds the nucleus. The probability of finding the electron (if we attempt to measure it) is greatest where the cloud is most dense.

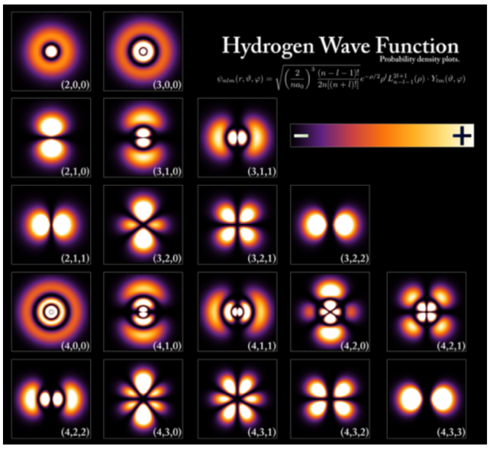

Figure 2. Atomic orbitals for different electron energy levels in hydrogen. The probability of finding the electron is given by the colour shown in the key (upper right).

RADIOACTIVE DECAY

CONTENT

Soon after the initial discovery of radioactivity, researchers found that applying an electric field would cause the radiation to split into three different types, as shown in Figure 1. This result indicated that alpha particles are positively charged, beta particles are negatively charged, and gamma rays have no electric charge.

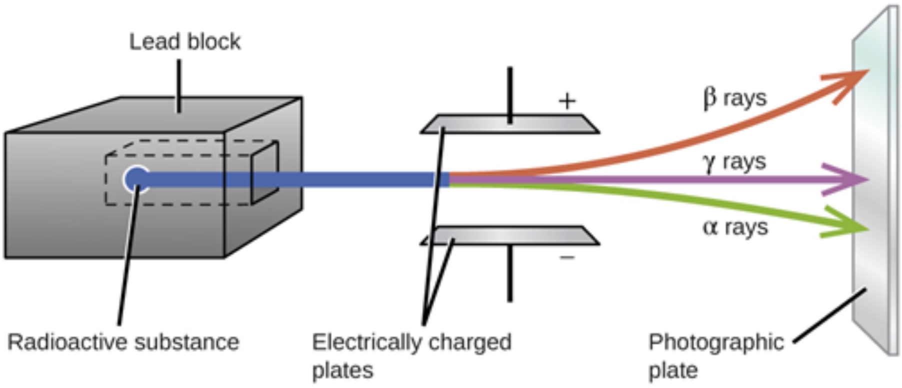

Figure 1. The discovery of three different types of radiation

Image Credit: http://letslearnnepal.com/class-12/physics/modern-physics/radioactivity

These types of radiation were named by their penetrating power, with alpha radiation being the least penetrating, being shielded by a few cm of air or just a sheet of paper. Beta radiation was found to be somewhat more penetrating, requiring a few mm of metal, such as aluminium in order to be blocked. Gamma rays are by far the most penetrating requiring several centimetres of lead in order to be absorbed. All of these are referred to as 'ionising radiation'. This means that the energy of this radiation is sufficient to ionise atoms and break chemical bonds. This is biologically important, as this is the type of radiation that can damage our DNA and cells, causing injury or even death. Examples of non-ionising radiation found in our everyday lives are microwaves and radio waves. While alpha radiation is the least penetrating it is actually the most ionising of these three forms of radiation. Although this may be counter-intuitive at first, it is the fact that the alpha particles so readily ionise the material they encounter, and transfer their energy into that material, that leads to them penetrating less material and dumping all of their energy in a small volume.

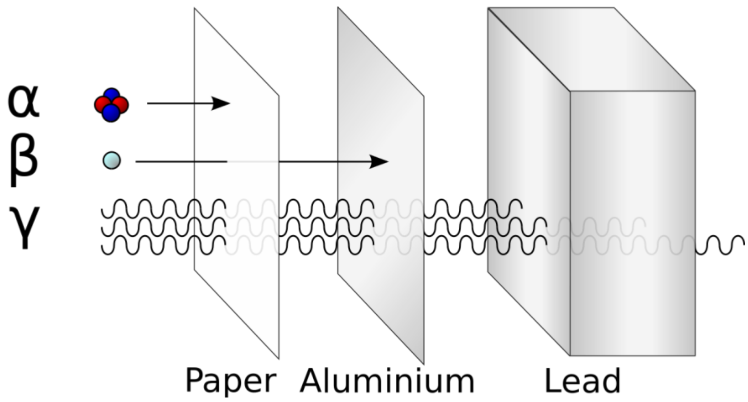

Figure 2. Penetrating power of alpha, beta, and gamma radiation
User: Stannered derivative work: Ehamberg/Wikimedia Commons/ CC BY 2.5

The composition of alpha, beta and gamma radiation is also different. An alpha particle is actually a helium-4 nucleus, which itself consists of two protons and two neutrons. Alpha decay typically occurs only in heavy nuclei. With the exception of beryllium-8 which spontaneously decays into two alpha particles, there are no elements below tellurium (atomic number 52) that undergo alpha decay. However, all elements heavier than lead (atomic number 82) are unstable, with the majority decaying through alpha radiation. Alpha decay has the largest effect on the decaying nucleus reducing the atomic mass number by 4 and atomic number by 2. A common isotope which is regularly used by students to observe an alpha decay is americium-241. Americium-241 has a relatively long half-life of 432.2 years, and the decay process is summarised in the equation below:

$$^{241}_{95}Am \rightarrow \, ^{237}_{93}Np + \, ^{4}_{2}\propto + \, \gamma \; 59 \; \mathrm{keV}$$

As you can see, this decay process also results in the emission of a gamma ray with an energy of 59 keV. Gamma rays always occur with another decay process. This is because the gamma rays are not actually emitted during the initial alpha or beta decay, but rather from the resultant nucleus. In the americium-241 example above the neptunium nucleus is actually created in an excited state, and this excited state rapidly decays (with a very short half-life) into a stable ground state with the emission of a gamma photon. This is very similar to how electrons that are bound to atoms can decay from excited states to lower energy states with the emission of a photon.

A beta particle is an electron that has been ejected from a nucleus. The electron is created in the nucleus of the atom by a neutron decaying into a proton and an electron. The proton remains in the nucleus and the electron is emitted. Beta decay typically occurs in nuclei that have a large ratio of neutrons to protons. Since beta decay will reduce the number of neutrons by one and increase the number of protons by one, the ratio of neutrons to protons is made even by this reaction. A common example of beta decay is caesium-137, and the reaction is summarised in the equation below:

$$^{137}_{55}Cs \rightarrow \, ^{137}_{56}Ba + \, ^{0}_{-1}\beta$$

For alpha, beta and gamma radiation it is impossible to predict when any specific nucleus will decay. That is why we describe these reactions as ‘probabilistic’, and we are only able to predict the average fraction of nuclei that will decay over a certain period of time. This introduces us to the characteristic time scale which we call ‘half-life’ ($t_{1/2}$). This is the time it takes for half of the nuclei in a sample to decay. If we consider the number of nuclei remaining after a period of many half-lives the amount of remaining nuclei follows an exponential decay, with half the nuclei remaining after one ‘half-life’, and half of that half (a quarter) remaining after another ‘half-life’, and so on. This is depicted diagrammatically in Figure 3. This relationship is described mathematically by:

$$N_t = N_0 e^{-\lambda t}$$

where N_t is the number of particles at time *t*, N_0 is the number of particles present at *t=0*, $\lambda = \frac{\ln(2)}{t_{1/2}}$ is called the decay constant, and $t_{1/2}$ is the half-life.

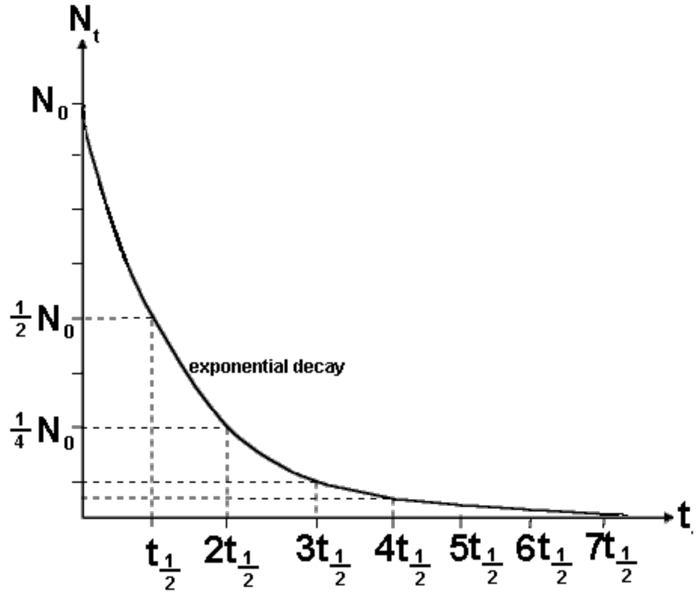

Figure 3. Number of nuclei versus number of half-life periods
Image Credit: https://www.cyberphysics.co.uk/topics/radioact/Radiodecay.htm

Example 1: In a student lab it is common to use americium-241 to demonstrate alpha decay. Say that a student's source contains 0.1g of americium-241. How much americium will be left after 1000 years? The half-life of americium-241 is 432.2 years.

$$N_t = N_0 e^{-\lambda t}$$

$$N_t = 0.1 e^{(-\frac{\ln(2)}{432.2})1000}$$

$$N_t = 0.020\ g$$

Example 2: Iodine-131 is a very dangerous substance with a short half-life (8.3 days), it is produced in significant quantities in nuclear reactors and weapons. Let's say a lake is polluted with iodine-131, how many days do we have to wait for 99.9% of the iodine to decay and for the water to be safe to drink?

$$N_t = N_0 e^{-\lambda t}$$

$$0.1 = 100 e^{-\lambda t}$$

$$1 \times 10^{-3} = e^{-\lambda t}$$

$$\lambda t = -\ln(1 \times 10^{-3})$$

$$t = \frac{-\ln(1 \times 10^{-3})}{\lambda}$$

$$t = \frac{-\ln(1 \times 10^{-3})}{(\frac{\ln(2)}{t_{1/2}})}$$

$$t = \frac{-\ln(1 \times 10^{-3})}{(\frac{\ln(2)}{8.3})}$$

$$t = 82.7\ days$$

$$t = 83\ days$$

FISSION AND FUSION

CONTENT

FISSION:

Nuclear fission can be described as either a nuclear reaction or a radioactive decay process. What defines this process is the splitting of the atom's nucleus into smaller, lighter nuclei. In the case of the fission of heavy elements, this process typically emits individual neutrons and gamma rays, and releases a very large amount of energy. It is the difference in mass between the original fuel nucleus and the resulting lighter nuclei that results in either the release or absorption of energy. Specifically, for fission to produce net energy, the resulting elements must have less mass than that of the original element. This difference in mass arises due to the difference in atomic "binding energy" between the atomic nuclei before and after the reaction (an example for this is given in the resource 'Nuclear Energy Calculations'). The energy contained in a nuclear fuel (e.g. ^{235}U, ^{239}Pu) is millions of times the amount of energy contained in a similar mass of chemical fuel. This makes nuclear fission an incredibly dense source of energy. It is nuclear fission which produces the energy of nuclear power plants and also drives the explosive force of nuclear weapons.

After a fission event, the resulting nuclei are no longer the same element as the original atom. Fission is, therefore, a type of nuclear transmutation. This process usually results in two nuclei of comparable sizes, but in around 0.2% of fission events three fragments are created. This special case is known as ternary fission. The fact that the products of fission reactions are not the same every time is a key difference between this and other types of radioactive decay, such as alpha and beta decay which give the same result in every decay reaction. These fission products are generally more radioactive than the original fuel elements and remain dangerously radioactive for long periods of time. This creates a significant issue for the safe disposal and storage of this nuclear waste. An additional issue is the diversion of nuclear fuel for use in weapons. These are the biggest roadblocks to more widespread use of nuclear energy.

The neutrons that are emitted during a fission event are emitted at a very high velocity, typically 7% the speed of light. A material is described as *fissionable* if it undergoes a fission reaction after absorbing one of these fast neutrons. It is preferable for reactor safety and efficiency that the fuel does not depend on these fast, neutron-induced fission reactions. It is preferable that the fuel is able to absorb much slower "thermal" neutrons to undergo a fission reaction. Such fuels are known as *fissile* materials. Some of the most commonly used fissile fuels are ^{233}U, ^{235}U and ^{239}Pu. An example of a fissionable element that is not fissile is ^{238}U, which is the most abundant form of uranium. It will undergo induced fission when impacted by a fast neutron. However, the average energy of the neutrons produced through the fission of ^{238}U is too low to induce further fission. For this reason, a chain reaction is not possible with ^{238}U.

A chain reaction of fission events can take place so long as the neutrons emitted from a fission event create at least one more fission event. Another way of thinking about this is also as the rate of generation of neutrons within the fuel versus the loss of neutrons from the fuel. As long as the rate of generation is greater a chain reaction will take place. A reactor which allows for a sustained nuclear chain reaction is commonly referred to as a critical assembly. In the case of a bomb, where the device is mainly just nuclear fuel, it is called a critical mass. An example of a sustained fission reaction is shown in Figure 1. Critical fission reactors are the most common type of nuclear reactor.

The fundamental physics of the fission chain reaction in a nuclear reactor is similar to a nuclear weapon. However, the two types of devices are very different in design. A nuclear weapon must release all its energy at once, while a reactor is designed to generate a steady supply of useful power. Put simply the main differences are the degree of criticality and the use of fast versus slow neutrons. In a reactor the ratio of neutrons produced in the fuel to those lost is only slightly greater than 1. Also, the time between the production of a neutron to its use in a fission event is on the order of tens of seconds. This results in a rate of power change of about 10% per second. This rate of change is slow enough that it can be controlled by reactor mechanisms. In a weapon, the time between the production of a neutron and its use in a fission event is much quicker, typically on the order of milliseconds. This results in a rate of power change of around 1,000,000% per second. This satisfies the purpose of the weapon, which is to create as much energy as fast as possible. Such a reaction would obviously not be able to be controlled in a reactor. Due to these and other design factors, it not possible for a nuclear reactor to explode with the same destructive power as a nuclear weapon.

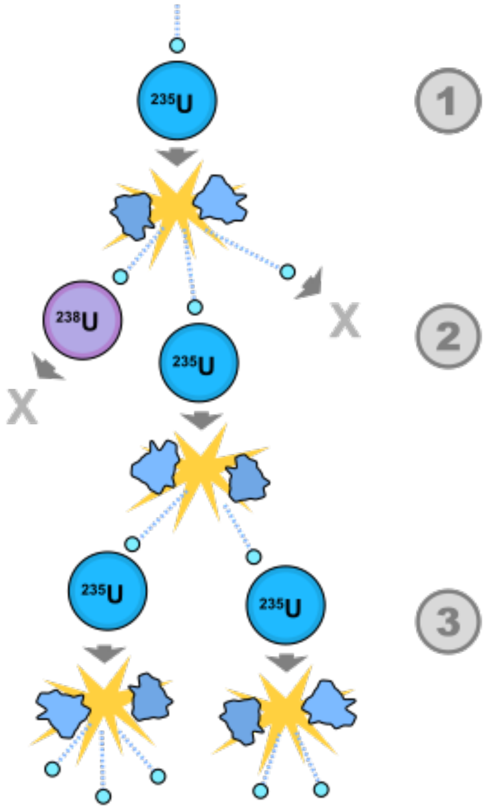

Figure 1: Diagram of a fission chain reaction

FUSION:

Nuclear fusion is a reaction in which two or more atomic nuclei are combined to form a number of different atomic nuclei and subatomic particles. As with fission, it is the difference in mass between the reactants and products which determines the amount of energy that is either released or absorbed. A fusion reaction that produces a nucleus **lighter** than iron will result in a net release of energy. The opposite is true for nuclear fission, where elements **heavier** than iron produce a net release of energy.

Hydrogen to helium fusion is the primary source of energy in all stars. Mid-size stars will also fuse helium into heavier elements such as oxygen, carbon, and nitrogen. Supermassive stars will even fuse elements all the way up to iron. Fusion to heavier elements does not occur in significant amounts as these processes take energy from the star, and when the fusion energy begins to wane the gravitational force collapses the star and if the star is massive enough a supernova results. The supernova is so energetic that elements heavier than iron are created in significant amounts and this is the main way that these heavier elements are created in the universe.

Nuclear fusion has the potential to provide us within an abundant source of energy without the long-lived nuclear waste issues that arise with nuclear fission. This is because the products of fusion reactions we might use for power generation are either stable or have comparatively short half-lives.

Unlike nuclear fission, where we were able to create controlled nuclear fission in a relatively short period of time following our initial discovery of the phenomenon, we have been unable to create controlled nuclear fusion with a net output of energy despite many decades of research. The main reason we have been unable to create controlled nuclear fusion is that it requires temperatures of millions of degrees Celsius. When materials reach these temperatures (or even at much lower temperatures) the electrons separate from the positive nucleus to form free electrons and ions. A material such as this is generally considered the fourth state of matter and is called plasma. Since these ions and electrons are charged particles, combinations of electric and magnetic fields are used to control this material. This is challenging due to the extremely high energy of the plasma particles. A high degree of confinement of this plasma is necessary as any contact with materials at normal temperatures will rapidly cool the plasma and stop any fusion from occurring. There are many different approaches to confining these fusion plasmas, and the most successful to date is a device known as the Tokomak. This is a doughnut-shaped device which confines a plasma in a toroidal magnetic field.

NUCLEAR ENERGY CALCULATIONS

CONTENT

The laws of conservation of mass and energy are two fundamental premises in physics which state that both mass and energy cannot be created or destroyed and are therefore conserved. In most cases it serves us well to consider separately that the total mass and total energy is conserved, as this is true in all mechanical and chemical processes. However, when it comes to nuclear processes a significant change in the amount of mass *and* energy in the system is observed. These reactions highlight the fact that the laws of conservation of mass and energy are not correct in all circumstances. It is more accurate for us to state that it is the total amount of mass *and* energy that is conserved, and that mass and energy can be transformed into one another using Einstein's most famous equation expressing the equivalence of mass and energy, $E = mc^2$.

BINDING ENERGY

Binding energy is the term given to the energy that holds together the individual nucleons (protons and neutrons) within the nucleus of an atom. This is quite a large amount of energy, which is a result of how close the protons in the nucleus are to each other. Recall that particles which have like charges will generate a repulsion force between them and this force increases with the inverse square of the distance between them. The source of the binding energy is a small fraction of the mass of the nucleons; this mass is converted to binding energy according to $E = mc^2$.

Example 1:
Let's take a look at the binding energy of the helium nucleus. The most common form of helium is known as ^{4}He which contains two protons and two neutrons. In order to calculate the magnitude of the binding energy in a single atom of helium we need to compare the mass of 2 individual protons and 2 individual neutrons, with the mass of a helium nucleus.

Looking up a few values we find that

Mass of proton = $1.6726219 \times 10^{-27}$ kg

Mass of neutron = 1.674929×10^{-27} kg

Mass of helium nucleus = $6.6464807 \times 10^{-27}$ kg

So, the difference in mass between the individual nucleons and the helium atom is:

$$(2 \times mass\ of\ proton + 2 \times mass\ of\ neutron) - (mass\ of\ helium\ nucleus)$$

$$(6.69510 \times 10^{-27}) - (6.64466 \times 10^{-27})$$

$$0.050441 \times 10^{-27}\ \text{kg}$$

We then use the mass energy equivalence to convert to energy:

$$E = mc^2$$

$$E = 0.050441 \times 10^{-27} \times (3 \times 10^8)^2$$

$$E = 4.5334 \times 10^{-12}\ \text{J}$$

This seems like a very small amount of energy, and it is, but remember that this is for only one atom, and there are *a lot* of atoms in the amounts of things that we are more familiar with. To put this in perspective there are about 1.5 x 10^{23} atoms in 1 gram of helium, which gives us a binding energy of 680MJ per gram of helium, and this is quite a lot of energy!

The binding energy is relevant to fusion and fission reactions as it is a requirement **that the products of the reaction be more stable than the reactants** for either of these reactions to release a net gain of energy.

Energy Released In Nuclear Reactions

Example 2:
Let us calculate the energy released in the fusion reaction between deuterium and tritium. This is the most promising reaction for a potential fusion power source. The reaction is summarised below.

$${}^2_1D + {}^3_1T \rightarrow {}^4_2He + {}^1_0n$$

Here, a tritium nucleus fuses with a deuterium nucleus to produce a helium nucleus and a free neutron. By comparing the difference in the masses of the products and reactants, we can use E = mc^2 to calculate the energy released in this reaction. We can find from the literature the mass for the elements of the reaction

Mass of Deuterium: 3.34 x 10^{-27} kg
Mass of Tritium: 5.01 x 10^{-27} kg

Mass of Helium: 6.65 x 10^{-27} kg
Mass of Neutron: 1.67 x 10^{-27} kg

Calculating the energy released in joules

$$Energy\ Released = (Mass\ of\ reactants - Mass\ of\ products) \times speed\ of\ light^2$$

$$= \left((mass_T + mass_D) - (mass_{He} + mass_n)\right) \times speed\ of\ light^2$$

$$= \left(\left((3.34 + 5.01) - (6.65 + 1.67)\right) \times 10^{-27}\right) \times 3.00 \times 10^8$$

$$= 2.8 \times 10^{-12}\ \text{J}$$

Nuclear reactions are typically described in units of MeV. In order to convert to MeV we must first divide by the charge of an electron, to convert from J to eV. Then divide by 10^6 to convert eV to MeV.

$$Energy\ Released = 17.5\ MeV$$

This is consistent with the documented value of 17.59 MeV. This same method can be used to determine the energy released in any nuclear transmutation, including naturally occurring radioactive decay (alpha, beta, and gamma) and fission.

BINDING ENERGY PER NUCLEON

Let us now consider the changes in binding energy in the deuterium-tritium reaction. We won't go through calculating the binding energy for all of these nuclei as we did in the last example, but we can easily look up the binding energies in order to compare them.

Values:

Binding energy for Deuterium, 2224 keV

Binding energy of Tritium, 8,481 keV

Binding energy of ^{4}He, 28300 keV

We can clearly see that the binding energy for the helium nucleus is much greater than the deuterium and tritium nuclei, which indicates that the helium nucleus is more tightly bound and hence is a more stable nucleus. For this reason, it is a more energetically stable configuration for the nucleons (proton and neutrons) to exist in than as a deuterium and tritium nucleus.

However, there is one more caveat in understanding the relationship between binding energy and stability. The number that truly represents stability is not the total binding energy of the nucleus, but the binding energy per nucleon. This number accurately represents stability as it represents how easy it is to pull the individual nucleons apart.

For example, consider a nucleus that has 100 nucleons and a total binding energy of 200,000 keV, this would be a binding energy per nucleon of 2,000 keV. Now consider a nucleus with just 10 nucleons and a total binding energy of 100,000 keV, this would be a binding energy per nucleon of 10,000 keV. Despite the fact that the larger nucleus has more total binding energy, it is the smaller nucleus that is actually more stable as it has 5 times more binding energy per nucleon and hence requires 5 times more energy to separate an individual nucleon.

If we were to calculate the binding energy per nucleon for our previous deuterium-tritium example, we can see that it is still greater for helium than deuterium and tritium, hence the reaction can occur and release energy.

Binding energy per nucleon of Deuterium, 1112keV

Binding energy per nucleon of Tritium, 2827keV

Binding energy per nucleon of ^{4}He, 7075keV

This brings us to the final figure which helps us understand binding energy. This graph plots the binding energy per nucleon for a range of different nuclei. On the left-hand side of the graph we can see that as the sizes of the nuclei increase, the binding energy per nucleon also increases. Hence, in this region it is possible for fusion reactions to occur and release energy. This is because the larger nuclei are more stable than the smaller reactants. However, on the right-hand side of the graph the opposite is true; these large nuclei are actually more stable and release energy when they split apart to make smaller nuclei (fission). This is the opposite process to fusion and is called fission. The most stable nucleus shown on this graph is ^{56}Fe.

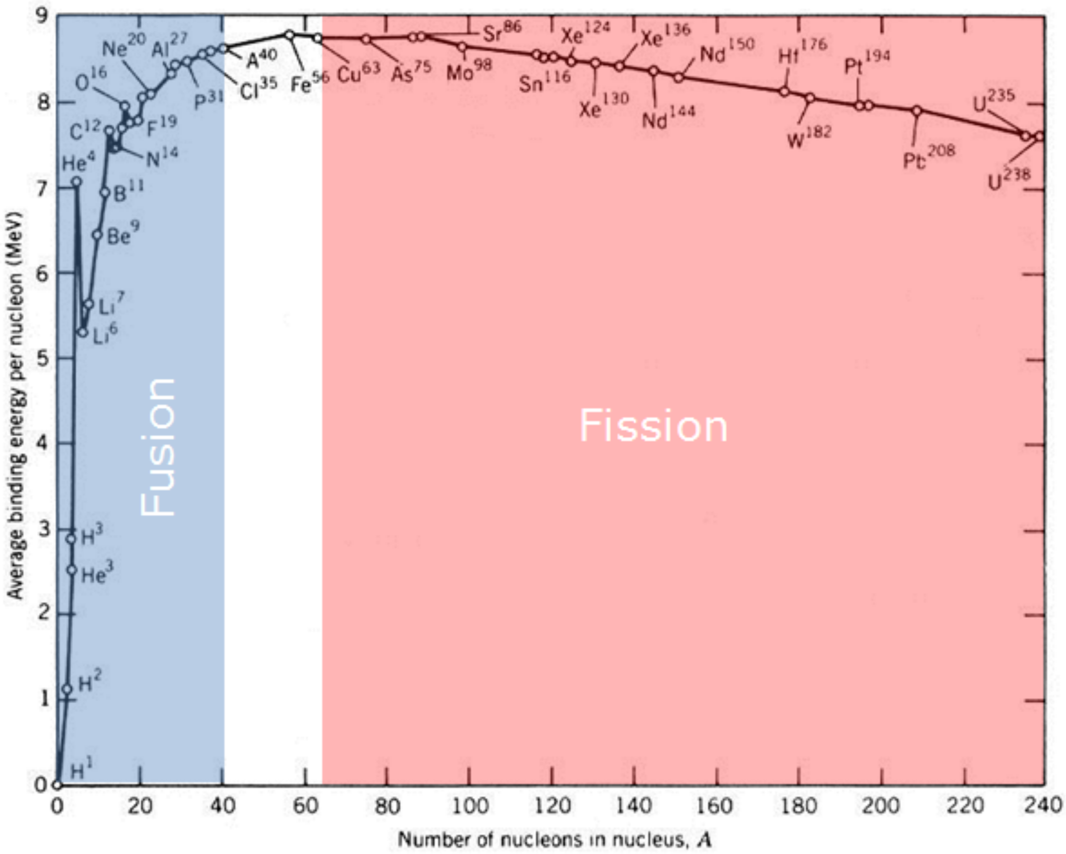

Figure 1. Binding energy per nucleon for a variety of atomic nuclei
Image Credit: http://www.earth-site.co.uk/Education/binding-energy/